THE VI CONFERENCE
OF MATHEMATICS AND COMPUTER SCIENCE

„CONGRESSIO-MATHEMATICA"

Olsztyn, Poland 21 – 22.11.2020
28 – 29.11.2020

Department of Complex Analysis
Faculty of Mathematics and Computer Sciences
University of Warmia and Mazury in Olsztyn
Sloneczna Street 54
10–710 Olsztyn
tel. 89 524 60 92

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Department of Complex Analysis
Faculty of Mathematics and Computer Sciences
University of Warmia and Mazury in Olsztyn

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Faculty of Technical Physics, Information Technology
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CONFERENCE PROGRAM

Saturday, November 21, 2020

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09:00 Opening

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Plenary lectures

Chairman: ***D. K. Thomas***

09:15 - 09:55	**J. Banaś**: *The technique of measures of noncompactness in the study of solutions of infnite systems of integral equations*
10:05 - 10:45	**M. Nowak**: *On kernels of Toeplitz operators*

Chairman: ***S. Singh***

10:55 - 11:35 (+4.5)	**V. Allu:** *The Bohr phenomenon for certain analytic and univalent functions*
11:45 - 12:25 (-1)	**D. K. Thomas**: *An overview of results on successive coeffcients of univalent functions*

Chairman: ***V. Allu***

12:35 - 13:15 (+4.5)	**S. Sivasubramanian:** *On a class of analytic functions related to Robertson's formula and subordination*
13:25 - 14:05 (+4.5)	**S. Singh:** *Zeros and geometric properties of hyper-Bessel functions*

Section - Complex analysis, I

Chairman: ***M. Mateljević***

14:15 - 14:40 (+4.5)	**S. Beig:** *Directional convexity of the convolution of harmonic mappings*
14:50 - 15:15 (+4.5)	**S. Sahoo:** *Bohr inequalities for certain integral operators*
15:25 - 15:50	**M. Svetlik:** *Some versions of the Schwarz lemma for harmonic mappings*
16:00 - 16:25	**N. Tuneski:** *Coeffcient inequalities for certain classes of univalent functions*

Chairman: ***A. Wiśnicki***

16:35 - 17:00	**I. Chyzhykov:** *Classes of analytic functions with prescribed set of singularities*
17:10 - 17:35	**A. Huczek:** *A unifed approach to Wolff-Denjoy's theorem*
17:45 - 18:10	**A. Ligęza:** *On Hamiltonians of the fourth Painleve equation*
18:20 - 18:45	**P. Michalak:** *Minimal generating set of directed unoriented Reidemeister moves*

Section - History and didactics of mathematics, I
(Organizers: S. Domoradzki and R. Długosz)

14:15 - 15:00 **I. Jóźwik, M. Terepeta:** *Matematycy Politechniki Łódzkiej w początkach istnienia uczelni*

15:10 - 15:40 **W. Walat:** *Główne problemy edukacji wyższej w związku z wprowadzeniem pełnego kształcenia zdalnego*

15:50 - 16:15 **M. Krukowski:** *Kilka słów o programujących matematykach*

16:25 - 16:50 **M. A. Zambrowska:** *Geometria w pierwszych latach szkolnej edukacji. Wybrane programy nauki z lat 1792-2020*

17:00 - 17:25 **R. Długosz, M. Lindner:** *Zwycięstwo technik synchronicznych*

17:35 - 18:00 **A. Szpila:** *Zdalne nauczanie w Uniwersytecie Rzeszowskim na kierunku matematyka – przebieg, problemy, efekty*

18:10 - 18:35 **W. Wójcik:** *The universal nature of Stefan Mazurkiewicz's mathematical research*

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Sunday, November 22, 2020

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Plenary lectures

Chairman: ***M. Nowak***

09:00 - 09:40 (+1) **D. Shoikhet:** *Nonlinear Resolvent and rigidity of holomorphic mappings*

09:50 - 10:30 (+7) **S. K. Lee:** *Zalcman conjecture*

Chairman: ***D. Shoikhet***

10:40 - 11:20 (+1) **M. Elin:** *Differentiability of semigroups of Lipschitz or smooth mappings*

11:30 - 12:10 (+4.5) **S. Kaliraj:** *Analytic and Harmonic Hardy Spaces*

Chairman: ***M. Elin***

12:20 - 13:00 **P. Liczberski, R. Długosz:** *A problem of the Fekete-Szegö type for Bavrin's families of holomorphic functions in* C^n

Chairman: ***P. Liczberski***

13:10 - 13:50 (+4.5) **A. Swaminathan:** *Geometric Properties of Analytic Functions Associated with Nephroid Domain*

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14:00-14:30 Poster session

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Plenary lectures

Chairman: ***M. Lachowicz***

14:35 - 15:15	**K. Kołowrocki:** *Safety analysis of multistate ageing system with inside dependencies and outside impacts*
15:25 - 16:05	**J. Chudziak:** *Positive homogeneity of the principle of equivalent utility*

Chairman: ***J. Chudziak***

16:15 - 16:55	**M. Golasiński:** *Harmonic polynomials and polynomial maps of spheres*
17:05 - 17:45	**M. Lachowicz:** *Integro-differential kinetic equations and this strange world*

Chairman: ***M. Golasiński***

17:55 - 18:35	**A. Wiśnicki:** *Linear and nonlinear extensions of the Ryll-Nardzewski theorem*

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Saturday, November 28, 2020

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Plenary lectures

Chairman: ***T. Sugawa***

09:00 - 09:40 (+1)	**A. Lyzzaik:** *Valency crietria for harmonic mappings of bounded boundary rotation*
09:50 - 10:15 (+1)	**D. Bshouty:** *Univalent Harmonic Mappings onto polygons. Topology and Geometry*

Chairman: ***A. Lyzzaik***

10:25 - 11:05 (+8)	**T. Sugawa:** *Coeffcient estimates of the Riemann mapping functions*
11:15 - 11:55	**M. Mateljević:** *On Kellogg's theorem for harmonic quasiconformal mappings*

Chairman: ***D. Bshouty***

12:05 - 12:45 (+4.5)	**S. Ponnusamy**: *The Bohr inequality for the generalized Cesàro averaging operators*
12:55 - 13:35	**A. Wolny-Dominiak:** *Risk premium in property/casualty insurance - statistical approach*

Chairman: ***S. Ponnusamy***

13:45 - 14:25	**J. Zając:** *Extremal problems for harmonic mappings with boundary normalization. Applications*
14:35 - 15:15	**D. Partyka:** *A simple deformation of harmonic mappings*

Chairman: ***J. Zając***

15:25 - 16:05 (-6)	**O. Ahuja:** *The Wonderful World of Quantum Calculus and Rapid Growth of the q-Disease in Geometric Function Theory*

Section - History and didactics of mathematics, II
(Organizers: S. Domoradzki and R. Długosz)

16:15 - 16:55	**M. Bečvářová:** *Women and Mathematics at the German University in Prague*
17:05 - 17:30	**P. Błaszczyk:** *Axioms for Euclid's theory of proportion*
17:40 - 18:05	**M. Zarichnyy:** *Topologia we wpisach z Księgi szkockiej*
18:15 - 18:40	**J. Koroński:** *Irena Łojczyk-Królikiewicz (1922-2019) w kontekście krakowskiej szkoły nierówności różniczkowych Jacka Szarskiego (1921-1980)*
18:50 - 19:15	**M. Wenderlich:** *Doświadczenia krystalizujące w rozwijaniu uzdolnień matematycznych polskich laureatów olimpiad międzynarodowych IMO*
19:25 - 19:50	**S. Domoradzki:** *Wspomnienie o śp. Dr Danucie Węglowskiej (1942-2020)*

Section - Applied mathematics

Chairman: ***K. Kołowrocki***

16:15 - 16:40	**B. Magryta:** *Safety and cost optimization of port and maritime transportation systems*
16:45 - 17:10	**E. Dąbrowska:** *Probabilistic approach to prediction of maritime oil spill movement*
17:15 - 17:40	**M. Torbicki:** *Modelling critical infrastructure safety impacted by climate-weather change process*
17:45 - 18:10 (+1)	**M. Kolev:** *On some applications of mathematical models in biology and medicine*

Chairman: ***M. Kolev***

18:20 - 18:45 (+1)	**I. Naskinova:** *Convolutional neural networks for X-ray chest images*
18:50 - 19:15 (+1)	**I. Nikolova:** *Analysis of autoimmune diseases by mathematical models*
19:20 - 19:45 (+1)	**B. Garkova:** *On the application of a mathematical model in electrochemistry*

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Sunday, November 29, 2020

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Plenary lectures

Chairman: ***S. K. Lee***

09:00 - 09:40 (+8)	**N. E. Cho:** *Various suffcient conditions for carathéodory functions*

Chairman: ***N. E. Cho***

09:50 - 10:30 (+4.5)	**V. Ravichandran:** *A survey on univalent functions with fixed second coeffcient*

Section - Complex analysis, II

Chairman: ***A. Swaminathan***

10:40 - 11:05 (+8)	**Y. J. Sim:** *Bounds for the fifth coefficients of analytic functions*
11:10 - 11:35 (+4.5)	**M. Sharma:** *Higher Order Differential Subordination for the functions with positive real part using Admissibility Techniques*
11:40 - 12:05 (+4.5)	**S. Anand:** *Certain estimates for a unified class of normalized analytic functions*

Chairman: ***V. Ravichandran***

12:15 - 12:40 (+4.5)	**S. Kumar:** *Differential subordination for certain Carathéodory functions*
12:45 - 13:10 (+4.5)	**K. Sharma:** *Radius of starlikeness for two classes of analytic functions*
13:15 - 13:40 (+4.5)	**A. Sebastian:** *Radius of starlikeness of certain classes of analytic functions*
13:45 - 14:10 (+4.5)	**S. Malik:** *Radius of starlikeness for Bloch functions*

Chairman: ***S. Sivasubramanian***

14:20 - 14:45 (+4.5)	**S. Kavitha:** *Coeffcient estimate results for few subclasses of analytic functions involving subordination*
14:50 - 15:15 (+4.5)	**L. Wani:** *Radius constants for functions associated with a limacon domain*
15:20 - 15:45 (+4.5)	**M. Ahamed:** *Uniqueness result of meromorphic functions and some shared value problems*

Chairman: ***D. Partyka***

15:55 - 16:20 (+2)	**A. Khalfallah:** *Some properties of mappings admitting general Poisson representations*
16:25 - 16:50	**I. Matychyn:** *On optimal control of linear fractional differential equations with variable coefficients*
16:55 - 17:20	**E. Trybucka:** *On a new result for a family of even holomorphic functions of several complex variables*
17:25 - 17:50	**M. Parol:** *Kaplan classes for polynomials with all zeros on unit circle*

Section - Computer science
(Organizer: P. Artiemjew)

Chairman: ***P. Artiemjew***

14:45 - 15:05	**A. M. Zbrzezny, A. Zbrzezny:** *Checking MTL Properties of Timed Automata with Dense Time using Satisfiability Modulo Theories*
15:10 - 15:30	**G. Białoskórska for A. Niemczynowicz, R. Kycia:** *Bayesian compression for categorical data)*
15:35 - 15:55	**K. Pancerz:** *Classification Strategies in Classification and Prediction Software System (CLAPSS*
16:00 - 16:20	**J. F. Peters:** *Amiable Fixed Sets. Extension of the Brouwer Fixed Point Theorem*

16:25 - 16:45	**Z. E. Csajbók.:** *Rough Continuity in the View of Intuitionistic Fuzzy Sets*
16:50 - 17:10	**S. K. Tadeja, D. Janik, P. Stachura:** *Object Assembly Assisted with Augmented Reality Interface and QR-Based Tagging : An Early Stage Report*
17:15 - 17:35	**B. Staruch, B. Staruch:** *Parametrized family of similarity-based classifiers*
17:40 - 18:00	**R. Kycia, A. Niemczynowicz:** *Landauer's principle in multivalued logic.*

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18:10 - 18:25 Closing

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ABSTRACTS

Om AHUJA

Department of Mathematical Sciences, Kent State University (Ohio, U.S.A)

The Wonderful World of Quantum Calculus and Rapid Growth of the $q-$Disease in Geometric Function Theory

The study of 300 years old history of quantum calculus or $q-$calculus, since the studies of Bernoulli and Euler, is often considered to be one of the most difficult subjects to engage in mathematics. Nowadays, there is a rapid growth of activities around $q-$calculus due to its applications in various fields such as mathematics, mechanics and physics. The history of study of $q-$calculus may be illustrated by its wide variety of applications in quantum mechanics, analytic number theory, theta functions, hypergeometric functions, theory of finite differences, gamma function theory, Bernoulli and Euler polynomials, Mock theta functions, Combinatorics, umbral calculus, multiple hypergeometric functions, Sobolev spaces, operator theory, and more recently in the theory of analytic and harmonic univalent functions. In $q-$calculus, we are generally interested in $q-$analogues that arise naturally, rather than in arbitrarily contriving $q-$analogues of known results. While focusing on excitement and romance with development of $q-$calculus and its applications, this presentation will also look at our recent results of $q-$calculus in the theory of analytic and harmonic univalent functions

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Vasudevarao ALLU

School of Basic Science, Indian Institute of Technology Bhubaneswar
(Bhubaneswar, India)

The Bohr phenomenon for certain analytic and univalent functions

The Bohr phenomenon for analytic functions of the form $f(z)=\sum_{n=0}^{\infty} a_n z^n$, first introduced by Harald Bohr in 1914, deals with finding the largest radius r_f, $0<r_f<1$, such that the inequality $\sum_{n=0}^{\infty}|a_n z^n| \leq 1$ holds whenever the inequality $|f(z)| \leq 1$ holds in the unit disk $\mathbf{D}=\{z\in\mathbf{C}:|z|<1\}$. The exact value of this largest radius known as Bohr radius, has been shown to be $r_f=1/3$. In this talk, we discus the Bohr radius for several analytic and univalent functions in the unit disk $\mathbf{D}$.

AMS Subject Classification: Primary 30C45, 30C50, 30C80
Keywords: Analytic, univalent, harmonic functions; starlike, convex, close-to-convex functions; coefficient estimate, growth theorem, Bohr radius.

Swati ANAND

Department of Mathematics, Univeristy of Delhi (Delhi, India)

Certain Estimates For a Unified Class of Normalized Analytic Functions

In this note, we investigate the distortion theorem and growth theorem for the unified class of normalized analytic functions which satisfy the second order differential subordination $f'(z) + \alpha z f''(z) \prec \phi(z)$, where ϕ is the analytic function with positive real part and normalized by the conditions $\phi(0) = 1$ and $\phi'(0) > 0$. Further, we compute an estimates on initial logarithmic coefficients, on inverse coefficient and on the second Hankel determinant involving the inverse coefficient for such functions.

Joint work with Naveen Kumar Jain and Sushil Kumar.

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Józef BANAŚ

Department of Nonlinear Analysis, Rzeszow University of Technology
(Rzeszów, Poland)

The technique of measures of noncompactness in the study of solutions of infinite systems of integral equations

A lot of real world problems (in engineering, mathematical physics, thermodynamics, economy etc.) can be described with help of integral equations and systems of integral equations. Particularly, some problems in question can be modelled with help of infinite systems of integral equations. In the present lecture we are focused on some considerations connected with infinite systems of integral equations. The theory of such systems is very young which is caused mainly by the fact that the tools allowing to investigate the solvability of mentioned infinite systems is not sufficiently developed up to now. The main difficulty in that theory depends on the fact that solutions of infinite systems of integral equations are created by sequences of functions defined, say, on some real interval, bounded or not.
Thus, in the study of the solvability of infinite systems of integral equations we are forced to establish a suitable space (Banach or Fréchet, for example) consisting of function sequences which

contains potential solutions of the mentioned infinite systems. Additionally, we have to choose suitable methods allowing us to obtain existence theorems for infinite systems integral equations.

In our talk we are going to present methods associated with the theory of measures of non-compactness which turn out to be very convenient and fruitful in obtaining theorems on the existence of solutions of infinite systems of integral equations. Moreover, our approach enables us to obtain the existence of solutions having some a priori imposed properties.

Martina BEČVÁŘOVÁ

Institute of Applied Mathematics, Faculty of Transportation Sciences
Czech Technical University in Prague (Prague, Czech Republic)

Women and Mathematics at the German University in Prague

In the first part, a short description of the historical background of women studies in Prague, resp. in Czech lands (later Czechoslovakia) will be given for a better understanding of some difficulties concerning the female studies at the secondary schools, universities and their doctoral procedures in mathematics at the German University in Prague during its whole existence (i.e. from 1882 until 1945).

In the second part, thanks the deep archival studies and primary sources, the detailed analysis of the successful doctoral procedures of three women graduated in mathematics at the German University in Prague will be presented with showing their life stories, pro-fessional activities, mathematical interests and results and fates of their families and relatives. We will talk about Saly Ruth Ramler married Struik (1894–1993), Hilda Falk (1897–1942) and Josefine Mayer born Keller (1904–1986).

Subzar BEIG

GDC Uri, University of Kashmir (India)

Directional convexity of the convolution of harmonic mappings

Recent developments on the convexity/directional convexity of the convolution of some univalent harmonic mappings including the right half-plane mappings, the vertical strip mappings and the square mappings will be discussed. Moreover, I will discuss the directional convexity of the convolution of harmonic mappings which are shares of odd starlike mappings or starlike mappings of order 1/2 and have real coefficients.

Gabriela BIAŁOSKÓRSKA[1] for Agnieszka NIEMCZYNOWICZ[2] and Radosław KYCIA[3]

[1]*Cracow University of Technology* (Cracow, Poland)
[2]*University of Warmia and Mazury in Olsztyn* (Olsztyn, Poland)
[3]*Masaryk University* (Brno, Czech Republic)
Cracow University of Technology (Cracow, Poland)

Bayesian compression for categorical data

We present a way to encode categorical data using the Bayesian approach. Due to the massive reduction of information, while maintaining statistical relationships, this method can be called a

Bayesian compression. Such representation can be used as an input for a Monte Carlo generator to increase data multiplicity. The implementation of these ideas in Python will be presented as well as its validation. Apart from compression, such a structure can be used to build models from data, feature extraction, and machine learning. Our results were inspired by the analysis of sociological data within the NAWA project 'The International Academic Partnership for Generation Z'.

Piotr BŁASZCZYK, Anna PETIURENKO

Pedagogical University of Krakow (Krakow, Poland)

Interpreting book VI of Euclid's Elements

Book VI of Euclid's *Elements*, through its consists of 33 propositions, develops the ancient theory of similar figures. We provide an axiomatic account of this book by appling the area method, as developed in (Chou et al. 1994). We also discuss an interpretation of book VI started by Hilbert and developed further by Hartshorne.

Our principal goal is to reconstruct, both the thesis (*protasis*) and the proof (*apodeixis*) of each proposition from VI.2 to VI.32. We show that Hilbert-Hartshorne account enables to recinstruct only Euclid's theses. The primitive concepts of the system we adopt are: point, length of directed segment, and signed area of triangle. They enable to interpret Euclid's line segments, triangles, and polygons with no reference to objects such as real numbers, *the measure of the area of the triangle* (Hilbert 1902, p. 62) or *the area of the figure* (Hartshorne 2000, p. 205). The basic proof technique applied in book VI is the theory of proportion, as developed in book V. We interpret proportions as fractions, still while transforming propotion we rely only on propositions of book V.

(Hilbert 1902) introduces proportions line segments, and a concept of the content of figuree. (Hartshorne 2000) applies these ideas to reconstruct book VI of the *Elements*. Equal content is a relation between two figures and it allows to interpret Euclid's theory of equal figures. Therefore, when interpreting book VI, Hartshorne can interpret terms standing for equality of figures proportions of line segments. However, he cannot interpret neither terms that include: proportions of figures and line segments (e.g. VI.2, 20), nor porportions of figures (e.g. VI.20, 25). Our axiomatic account enables to reconstruct propositions VI.2 through V.32. Proposition VI.31 we adopt as an axiom. With regard to proposition V.33, we reveal an inherent inconsistency of Euclid's theory of proportion when applied to angles.

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Daoud BSHOUTY

Technion - Israel Institute of Technology (Haifa, Israel)

Univalent Harmonic Mappings onto polygons. Topology and Geometry

Univalent harmonic mappings onto polygons were first studied by Sheil-Small in 1989. They may be called Sheil-Small mappings. Ever since, the vast information that we know today about the theory of harmonic mappings, other fields infiltrated into the field. I am going to discuss some geometric and topological methods in this direction and use it to give a counterexample to a conjecture.

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Nak Eun CHO

Pukyong National University (Busan, Korea)

Various Sufficient Conditions for Carathéodory Functions

The purpose of the present talk is to provide various sufficient conditions for Carathéodory functions and strongly starlike functions by using the results given by Miller and Mocanu, and Nunokawa. We also give some applications to geometric function theory as special cases of the main results presented here.

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Zoltán Ernő CSAJBÓK

Department of Health Informatics, Faculty of Health, University of Debrecen
Sóstói út 2-4, H-4406 Nyíregyháza (Hungary)
csajbok.zoltan@foh.unideb.hu

Rough Continuity in the View of Intuitionistic Fuzzy Sets

In the mid 1990s Z. Pawlak relying on the rough set theory (RST) [12, 13, 18] originated the study of rough calculus in his many papers [14, 15, 16, 17]. Its initial notions are the following.

A *categorization* of the interval $I = [0, a]$[1] is a strictly monotone sequence $S_I = \{x_i\}_{i\in[n]} \subseteq \mathbf{R}^{\geq 0}$, where $n \geq 1$ and $0 = x_0 < x_1 < \ldots < x_n = a$. Elements of S_I are called *categorization points* of I.

Let I_S denote the equivalence relation which is generated by S_I: xI_Sy if $x = y = x_i \in S_I$ for some $i \in [n]$[2] or $x, y \in \,]x_i, x_{i+1}[$ for some $i \in [n[$. Hence, the partition I/I_S associated with the equivalence relation I_S is $I/I_S = \{\{x_0\},]x_0, x_1[, \{x_1\}, \ldots, \{x_{n-1}\},]x_{n-1}, x_n[, \{x_n\}\}$.

The block containing $x \in I$ is denoted by $[\![x]\!]_{I_S}$. In particular, if $x \in S_I$, $[\![x]\!]_{I_S} = \{x\}$. If $x \in [\![x]\!]_{I_S} = \,]x_i, x_{i+1}[\,$, then $\overline{[\![x]\!]}_{I_S}$ denotes the closed interval $[x_i, x_{i+1}]$.

Let us define the following numbers: $\mathsf{l}_S(x) = \max\{x' \in S_I \mid x' \leq x\}$, $\mathsf{u}_S(x) = \min\{x' \in S_I \mid x' \geq x\}$.

---

[1] Let $a, b \in \mathbf{R}$ $(a \leq b)$. $[a, b] = \{x \in \mathbf{R} \mid a \leq x \leq b\}$ and $]a, b[= \{x \in \mathbf{R} \mid a < x < b\}$ denote the closed and open bounded intervals. It is easy to interpret, then, the open-closed $]a, b]$ and closed-open $[a, b[$ intervals.

[2] $[n] = \{0, 1, \ldots, n\} \subset \mathbf{N}$ is a finite set of natural numbers. Accordingly, $]n] = \{1, \ldots, n\}$, $[n[= \{0, 1, \ldots, n-1\}$ and $]n[= \{1, \ldots, n-1\}$.

The number $x \in I$ is *exact* if $\mathsf{l}_S(x) = \mathsf{u}_S(x)$, otherwise x is *inexact* or *rough* [16]. Of course, $x \in I$ is exact iff $x \in S_I$. In this context, the members of I/I_S are called the *rough numbers.* In addition, the categorization points in S_I are called the *roughly isolated points.*

Let $I = [0, a_I]$ and $J = [0, a_J]$ be two intervals with $a_I,\ a_J \in \mathbf{R}^{\geq 0}$, $a_I, a_J > 0$. Let S_I, P_J be the categorizations of I and J, where $S_I = \{x_i\}_{i\in[n]}$ and $P_J = \{y_j\}_{j\in[m]} \subseteq \mathbf{R}^{\geq 0}$ in such a way that $m, n \geq 1$, and $0 = x_0 < x_1 < \cdots < x_n = a_I$, $0 = y_0 < y_1 < \cdots < y_m = a_J$.

A Cartesian coordinate system whose x and y axes equipped with S_I, P_J categorizations is called the (S_I, P_J)*–coordinate system* or *rough coordinate system* for short. Any function $f \in J^I$ attached to a rough coordinate system is called the *rough real function.*[1]

After all, the question is how calculus–like notions, mainly such as continuity, differentiation and integration can be defined concerning rough real functions.

In my talk, first, I survey my results concerning rough continuity by the papers [7, 10]*. Then, I present some necessary and sufficient conditions for the rough continuity in terms of intuitionistic fuzzy set theory terminology based on the papers* [8, 9, 11].

Rough continuity is just as the central notion in rough calculus as the continuity in classical real analysis.

Let I and J two real intervals with categorizations S_I and P_J as they are given above.

Definition 1 ([16])**.** A rough real function $f \in J^I$ is (S_I, P_J)*–continuous* or *roughly continuous* at $x \in I$ if $f(\llbracket x \rrbracket_{I_S}) \subseteq \llbracket f(x) \rrbracket_{J_p}$. Otherwise, f is (S_I, P_J)*–discontinuous* or *roughly discontinuous* at x.

f is (S_I, P_J)*–continuous* (*roughly continuous*) on $I' \subseteq I$ if f is (S_I, P_J)–continuous at every point of I'. Otherwise, f is *not roughly continuous on* I'.

Proposition 2. *A rough real function $f \in J^I$ is (S_I, P_J)–continuous at every $x \in S_I$ roughly isolated point.*

Definition 3 ([10])**.** The (S_I, P_J)-discontinuity of $f \in J^I$ is called

(1) the *rough jump discontinuity of the first kind* if it is derived from touching a straight line segment $y = y_j$ for some $j \in [m]$;
(2) the *rough jump discontinuity of the second kind* if it is derived from intersecting a straight line segment $y = y_j$ for some $j \in]m[$;
(3) any other type of discontinuity is called the *rough jump discontinuity of the third kind.*

Proposition 4. *A rough real function $f \in J^I$ is (S_I, P_J)–continuous on I if and only if f does not have rough jump discontinuity of any kind.*

In order to establish a connection between the rough real functions and the intuitionistic fuzzy sets, the rough real functions must first be represented.

Definition 5 ([7, 8])**.** Let $f \in J^I$. The *block by block, blockwise in short,* (S_I, P_J)*–lower* and (S_I, P_J)*–upper approximations* of f are the functions

$$\underset{\leftrightarrow}{f} : I \to P_J,\ x \mapsto \mathsf{l}_P(\inf f(\llbracket x \rrbracket_{I_S})), \quad \overset{\leftrightarrow}{f} : I \to P_J,\ x \mapsto \mathsf{u}_P(\sup f(\llbracket x \rrbracket_{I_S})).$$

f is *blockwise exact* on a block if its direct images with respect to $\underset{\leftrightarrow}{f}$ and $\overset{\leftrightarrow}{f}$ are equal; otherwise f is *blockwise inexact* (*rough*) on it. f is *blockwise exact on* I if it is blockwise exact on every block; otherwise f is *blockwise inexact* (*rough*) *on* I.

[1] Let U, V be two nonempty sets. A function f is denoted by $f : U \to V$, $u \mapsto f(u)$ with domain $\mathsf{Dom} f = U$ and co-domain $\mathsf{Im} f = V$. In addition, $u \mapsto f(u)$ is the assignment or mapping rule of f. For any $S \subseteq U$, $f(S) = \{f(u) \mid u \in S\} \subseteq V$ is the direct image of S. V^U denotes the set of all such functions.

$\underset{\leftrightarrow}{f}$ and $\overset{\leftrightarrow}{f}$ are the *blockwise representation* of f [7]. Although, they are defined point by point, they are constant on every block, and so, the use of the phrase “blockwise” is justifiable.

According to Definition 5, $\underset{\leftrightarrow}{f}, \overset{\leftrightarrow}{f} \in [0,1]^I$, that is, they are fuzzy sets. Moreover, it is easy to check that $\underset{\leftrightarrow}{f} \leq \overset{\leftrightarrow}{f}$ also holds. Hence, $f_{bw}^{IVFS} = \left[\underset{\leftrightarrow}{f}, \overset{\leftrightarrow}{f}\right]$ forms an interval–valued fuzzy set, and so the function pair $f_{bw}^{IFS} = \left(\underset{\leftrightarrow}{f}, 1 - \overset{\leftrightarrow}{f}\right)$ is an intuitionistic fuzzy set [1, 6].

In terms of intuitionistic fuzzy set theory, $\underset{\leftrightarrow}{f}$ and $1 - \overset{\leftrightarrow}{f}$ are the IFS *membership* and *non-membership functions*, $\pi_{\overset{\leftrightarrow}{f}} = 1 - \underset{\leftrightarrow}{f} - (1 - \overset{\leftrightarrow}{f}) = \overset{\leftrightarrow}{f} - \underset{\leftrightarrow}{f}$ is the IFS *indeterminacy function* [2, 3, 4].

The intuitionistic fuzzy set f_{bw}^{IFS} is derived from f with respect to a $(S_I, P_{[0,1]})$–coordinate system. It is called the *blockwise roughly derived intuitionistic fuzzy set.*

Definition 6 ([8])**.** A blockwise roughly derived IFS f_{bw}^{IFS} is *roughly strong* if $f_{bw}^{IFS} = g_{bw}^{IFS}$ for every such function $g \in J^I$ that $g = \underset{\leftrightarrow}{f} = \overset{\leftrightarrow}{f}$ on the blocks where f is blockwise exact, and $\underset{\leftrightarrow}{f} < g < \overset{\leftrightarrow}{f}$ on the blocks where f is blockwise inexact.

Theorem 7 ([8])**.** *A rough real function $f \in J^I$ is (S_I, P_J)–continuous on I if and only if the blockwise roughly derived IFS f_{bw}^{IFS} is roughly strong.*

Intuitionistic fuzzy representation of rough real functions gives a opportunity to apply some intuitionistic fuzzy methods to them. For instance, rough real functions can be compared and ranked [5, 19], their distances can be determined [20, 21], the amount of knowledge included in them can be measured [22].

The above outlined method can be generalized in different ways. Intervals on x and/or y axes can be covered overlapping intervals instead of partition; instead of line segments, curve segments can be assigned to categorization points.

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Jacek CHUDZIAK

University of Rzeszów (Rzeszów, Poland)

Positive homogeneity of the principle of equivalent utility

Insurance contract pricing consists on assigning to any risk a non-negative real number, interpreted as a premium for insuring the risk. Risks are usually represented by non-negative bounded random variables on a given probability space. There are several methods of insurance contract pricing. Some of them are based on explicit formulas, others are defined in an implicit way. In this talk we deal with the principle of equivalent utility, introduced by Bühlmann [1]. The principle, involving the notion of a utility function, postulates a fairness in terms of utility.

Recently several authors have investigated some properties of the principle of equivalent utility under various behavioral models of decision making under risk. In particular, Kałuszka and Krzeszowiec [2] introduced and investigated the principle under the cumulative prospect theory. It turns out that the principle is monotone and translative. Another important and desirable property of the principle is positive homogeneity, which however usually does not hold. In the talk we are going to present a characterization of that property.

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Ewa DĄBROWSKA

Gdynia Maritime University (Gdynia, Poland)

Probabilistic approach to prediction of maritime oil spill movement

The probabilistic approach and based on it the supplemented Monte Carlo simulation method to the oil spill domain movement investigation is proposed. A two-dimensional stochastic process is used to describe the oil spill domain central point position. A stochastic model of the process of changing hydro-meteorological conditions is constructed. Parametric equations of oil spill domain central point drift trend curve for different kinds of hydro-meteorological conditions are determined. Monte Carlo prediction procedure and an algorithm of oil spill domain movement at varying in time hydro-meteorological conditions is created.

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Renata DŁUGOSZ[1], Piotr LICZBERSKI[2]

[1] *Centre of Mathematics and Physics, Lodz University of Technology* (Łódź, Poland)
[2] *Institute of Mathematics, Lodz University of Technology* (Łódź, Poland)

A problem of the Fekete-Szegö type for Bavrin's families of holomorphic functions in $\mathbf{C}^n$

Bavrin in the paper [1] considered some families of holomorphic functions in two-circle domains in $\mathbf{C}^2$ which are bounded, having positive real part and which Temljakov transform Lf has positive real part, respectively. For such functions he gave some estimates of the modulus of m-homogeneous polynomials occurred in power series expansion of functions.

During the lecture we will give for the above families a kind sharp estimate for the pair of homogeneous polynomials $Q_{f,2}, Q_{f,1}$, i.e., sharp estimate

$$\mu_{\mathcal{G}}\left(Q_{f,2} - \lambda\left(Q_{f,1}\right)^2\right) \leq M(\lambda), \quad \lambda \in \mathbf{C},$$

where $\mu_{\mathcal{G}}$ is the Minkowski balance of 2-homogeneous polynomials. The estimate is a generalization of a solution of the well known Fekete-Szegö coefficient problem in complex plane [2] onto the case of several complex variables.

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Renata DŁUGOSZ, Monika LINDNER

Centre of Mathematics and Physics, Lodz University of Technology (Łódź, Poland)

The victory for synchronous techniques

During the lecture we will present our experience with doing maths lessons on Lodz University of Technology in the circumstances of forced e-learnig.

Anna DOBOSZ, Adam LECKO

University of Warmia and Mazury in Olsztyn (Olsztyn, Poland)

Some generalized Briot-Bouquet differential subordination

A generalized Briot-Bouquet differential subordination is presented. Applications in the theory of ordinary differential equations are discussed also.

Stanisław DOMORADZKI

Uniwersytet Rzeszowski (Rzeszów, Poland)

Wspomnienie o śp. Dr Danucie Węglowskiej (1942-2020)

Danuta Węglowska ukończyła studia matematyczne na UJ w 1964 r. i od tegoż roku związała się z AGH w Krakowie, gdzie prowadziła zajęcia do 2020 r. Jest współredaktorem naukowym Słownika Biograficznego Matematyków Polskich (2003), przygotowanie którego zajęło redaktorom kilkanaście lat pracy. Współorganizowała, bądź efektywnie pomagała w organizacji wybranych Szkół Historii Matematyki, które zainicjowała dr Zofia Pawlikowska-Brożek również z AGH. Publikowała z historii matematyki. Była wybitnym i niezwykle odpowiedzialnym wykładowcą. Współpracowała z Komisją Historii Matematyki PTM działającą przy Zarządzie Głównym Towarzystwa. W referacie wspomniane zostaną jej najważniejsze dokonania.

Literatura

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Mark ELIN

ORT Braude College (Karmiel, Israel)

Differentiability of semigroups of Lipschitz or smooth mappings

In this talk we discuss the following problem: What conditions on the semigroup elements entail its differentiability with respect to t? We establish a sufficient condition for the differentiability of semigroups of nonlinear self-mappings of a domain in a Banach space that are Lipschitz or smooth with respect to the spatial variable. In this way we generalize the well-known result of Reich and Shoikhet (see for example, [2, 3]) on the differentiability of semigroups consisting of holomorphic self-mappings.

It turns out the problem is closely connected to geometry of the domain. At the end of the talk, we present a conjecture inspired by the classical work [1] by Bochner and Montgomery.

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Marek GOLASIŃSKI

University of Warmia and Mazury in Olsztyn (Olsztyn, Poland)

Harmonic polynomials and polynomial maps of spheres

Given a field K, we write $K[X_1, \ldots, X_n]$ for the ring of polynomials over K in variables $X_1, \ldots, X_n$. A map $p = (p_1, \ldots, p_n) : K^m \to K^n$ is called *polynomial* if $p_1, \ldots, p_n : K^m \to K$

are polynomial functions. The subset

$$\mathbf{S}^n(K) = \{(x_0, \dots, x_n) \in K^{n+1};\ x_0^2 + \dots + x_n^2 = 1\}$$

of K^{n+1} is called the *n-sphere* over K.

We say that a map $p = (p_0, \dots, p_n) : \mathbf{S}^m(K) \longrightarrow \mathbf{S}^n(K)$ is *polynomial* if it is a restriction of a polynomial map $P : K^{m+1} \to K^{n+1}$.

This talk aims to present a few comments on harmonic polynomials from $\mathbf{R}[X_1, X_2]$ and on representing homotopy classes of maps $\mathbf{S}^m(K) \to \mathbf{S}^n(K)$ by polynomial maps for $K = \mathbf{R}, \mathbf{C}$, the fields of reals and complex numbers.

First, we show that

Proposition 1. *A polynomial* $p \in \mathbf{R}[X_1, X_2]$ *is harmonic if and only if* $p = \operatorname{Re} P$, *the real part of some polynomial* $P \in \mathbf{C}[Y]$ *for* $Y = X_1 + \mathrm{i}X_2$.

If $K = \mathbf{R}$ then we write $\mathbf{S}^n(K) = \mathbf{S}^n$ and notice that $\mathbf{S}^1 = \{z \in \mathbf{C};\ |z| = 1\}$.

Proposition 2. *A map* $p = (p_0, p_1) : \mathbf{S}^1 \to \mathbf{S}^1$ *is polynomial if and only if* $f(z) = \alpha z^n$ *for* $z \in \mathbf{S}^1$ *with some* $n \geq 0$ *and* $\alpha \in \mathbf{C}$. *If* $(X_1 + \mathrm{i}X_2)^n = p_n + \mathrm{i}q_n$ *then polynomials* $p_n, q_n \in \mathbf{Z}[X_1, X_2]$ *over integers* $\mathbf{Z}$ *are harmonic.*

Remark 3. Further properties of the sequences (p_n) and (q_n) mimic those of Fibonacci numbers, Mersenne numbers and others.

Theorem 4. (1) *If* n *is odd then the element of the infinite cyclic homotopy group* $\pi_n(\mathbf{S}^n)$ *corresponding to the integer* k *can be represented by a form of degree* $|k|$ *mapping* $\mathbf{S}^n$ *to* $\mathbf{S}^n$*;*

(2) *if* n *is a power of* 2 *then all polynomial maps from* $\mathbf{S}^n$ *to* $\mathbf{S}^{n-1}$ *are constant.*

It follows from the above that all polynomial maps sending $\mathbf{S}^m \to \mathbf{S}^n$ are constant if $m \geq 2n$.

In view of the homotopy equivalence $\mathbf{S}^n(\mathbf{C}) \simeq \mathbf{S}^n$, we may state:

Theorem 5. (1) *If an element in* $\pi_{m-1}(\mathbf{S}^{2n-1})$ *is representable by a real homogeneous polynomial map of spheres* $p : \mathbf{S}^{m-1} \to \mathbf{S}^{2n-1}$ *then its suspension is representable by a complex polynomial map of* $\mathbf{S}^m(\mathbf{C}) \to \mathbf{S}^{2n}(\mathbf{C})$*;*

(2) *every element in* $\pi_n(\mathbf{S}^n)$ *can be represented by a complex polynomial map of* $\mathbf{S}^n(\mathbf{C})$ *for all positive integers* n.

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Aleksandra HUCZEK

Pedagogical University of Krakow (Cracow, Poland)

A unified approach to Wolff-Denjoy's theorem

The classical Wolff-Denjoy theorem states that if $f : \Delta \to \Delta$ is a holomorphic mapping of the unit disc $\Delta \subset \mathbf{C}$ without a fixed point, then there is a point $\xi \in \partial\Delta$ such that the iterates f^n converge locally uniformly to ξ on Δ. This theorem has been generalized over the years in different directions (see e.g. [1, 5, 6]). One of the researchers was Beardon [2] who, in particular, studied the Wolff-Denjoy type theorem for contractive mappings on strictly convex domains in Hilbert metric spaces [3].

The aim of this talk is to extend Beardon's framework. We present generalizations of the Wolff–Denjoy theorem for fixed-point free nonexpansive (i.e. 1-Lipschitz) mappings in proper geodesic spaces. In particular, we receive as special cases a few previous results for strictly convex bounded domains in $\mathbf{R}^n$ or $\mathbf{C}^n$ with respect to a large class of metrics including the Kobayashi, Hilbert and Thompson metrics.

The talk is based on joint work with Andrzej Wiśnicki.

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Izabela JÓŹWIK, Małgorzata TEREPETA

Politechnika Łódzka (Łódź, Poland)

Matematycy Politechniki Łódzkiej w początkach istnienia uczelni

W tym roku minęło 75 lat od powstania Politechniki Łódzkiej. Wieloletnie starania o utworzenie w Łodzi uczelni technicznej zostały uwieńczone sukcesem 24 maja 1945 roku. W roku akademickim 1945/46 Politechnika Łódzka obejmowała 3 wydziały: Mechaniczny, Elektryczny, Chemiczny oraz Oddział Włókienniczy, a wśród zatrudnionych profesorów byli matematycy: Witold Pogorzelski oraz Edward Otto. Program studiów na wszystkich wydziałach był 4-letni i wykłady z matematyki były wspólne dla wszystkich studentów.

W wystąpieniu chcemy przedstawić sylwetki matematyków, którzy pracowali na Politechnice Łódzkiej w ciągu pierwszych 10 lat istnienia uczelni.

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A. Sairam KALIRAJ

Department of Mathematics, IIT Ropar (India)

Analytic and Harmonic Hardy Spaces

In 1915, G.H. Hardy published one of his seminal research article entitled "*The mean value of the modulus of an analytic function*", which motivated F. Riesz to formally define the so called

"Hardy Spaces". Considering the applications of Hardy spaces, people started studying several other function spaces as well. In this talk, we shall discuss about some of the applications of Hardy spaces techniques in solving problems of univalent analytic functions and harmonic mappings in the plane. In particular, we discuss about the Riesz - Fejér inequality for complex-valued harmonic functions in the harmonic Hardy spaces $\mathbf{h}^p$ for all $p > 1$.

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S. KAVITHA

Department of Mathematics, SDNB Vaishnav College for Women
Chromepet, Chennai-600064 (Tamilnadu, India)

Coefficient estimate results for few subclasses of analytic functions involving subordination

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in \mathbf{D}, \tag{0.1}$$

which are *analytic* in the open unit disk $\mathbf{D} = \{z \in \mathbf{C} : |z| < 1\}$. Further, denote by $\mathcal{S}$, the subclass of $\mathcal{A}$ consisting of functions that are analytic and *univalent* in $\mathbf{D}$. In the present investigation, the author introduce a new class of functions that are analytic and univalent in the open unit disk and are subordinated to an univalent functions. For this new class, the author obtain lower and upper bounds for the second and third order Hermitian Toeplitz determinants for some analytic functions related to an univalent function by subordination.

Key Words and Phrases: Analytic, univalent, subordination.

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Adel KHALFALLAH

King Fahd University of Petroleum and Minerals (Dhahran, Saudi Arabia)

Some properties of mappings admitting general Poisson representations

The aim of this paper is twofold. Firstly, we adapt the Burgeth's spherical cap method [2, 3] to the planar case in order to establish some Schwarz type lemmas for mappings admitting general Poisson type representations on the unit disk. Secondly, we investigate some properties of generalized harmonic functions, called T_α-harmonic, introduced by Olofsson [8], and we prove a Heinz-Hethcote theorem [1, 5, 6, 7] and Landau type theorem for this class of functions, when $\alpha > 0$, see [4].

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Krzysztof KOŁOWROCKI

Gdynia Maritime University (Gdynia, Poland)

Safety analysis of multistate ageing system with inside dependencies and outside impacts

An innovative approach and a significant for real practical applications theoretical result are proposed for the safety analysis of multistate ageing systems that considers their components' dependency and their operation processes impacts. A safety function is defined and determined for a multistate ageing complex system with dependent components impacted by its operation process. As a special case, the safety of a multistate ageing homogeneous series system is modelled using its components' piecewise exponential safety functions. The results are applied to examine and characterize safety of an exemplary car wheel system impacted by its wheels' dependency and its operation process. Since there are no science and real practice in safety examination of the complex systems without considering their ageing, inside dependences and outside impacts, an innovative approach and significant and breakthrough for real practical applications new theoretical results are proposed for the safety analysis of multistate and ageing systems [9], [11], [15]-[16], [18]-[19], [26]-[31] that consider their components' dependency [1]-[2], [10], [23] and changing their safety parameters caused by their operation conditions [4], [9], [12]-[15], [17]-[22].

A safety function is defined and determined for a multistate ageing complex system and particularly for a multistate ageing series system [15] with dependent components [1]-[2], [10] and impacted by its operation conditions [21]-[22]. Other practically important system safety indicators are proposed as well. As a special case, the safety of a series system is modelled assuming its components piecewise exponential safety functions. The results are applied to examine and characterize a car wheel system safety. This practically important approach to multistate system safety analysis considers the assumption about components' degradation through departures from the safety state subsets impacted by their dependency and by the changing the system operation process as in real technical systems, components often degrade with different intensity of degradation at various system operation states. The paper is devoted to system ageing, its components inside system dependency and its outside operation impacts and to consider them together in system safety analysis and to show the possibility of this approach real application in practice. The paper is organized into 6 parts, this Introduction as Section 1, Sections 2-5 and Summary as Section 6. In Section 2, the multistate approach to ageing system safety analysis is introduced and the safety indicators of multistate ageing systems are defined. In Section 3, the safety of a multistate ageing system composed of dependent components and impacted by its operation process is considered. The safety function of the complex multistate ageing system with dependent components related to the system operation process impact is proposed as a main system safety indicator. In Section 4, a new result in the form of a proposition on safety of homogeneous multistate ageing series system simultaneously impacted by its components dependency and its operation process in the particular case when the system components have piecewise safety functions is formulated and justified. The applique Section 5, is devoted to the safety analysis of the car wheel system. The results from Section 4 are applied to determination safety and resilience indicators of the car wheel system with dependently ageing wheels and impacted by its operation process. In Summary, the results' evaluation is done and and the perspective for future research in the field considered in the paper is given. Combining the results of the safety analysis of multistate ageing systems with dependent components [10] and the results of the safety analysis of multistate ageing systems impacted by their operation processes [4], the joint safety analysis of a series multistate ageing system considering simultaneously its inside dependences and outside influences is performed and the new result that improves significantly the accuracy of the real system safety examination is found. As a consequence of the achieved new results, the further research could be focused on safety analysis of multistate ageing complex systems [15] and critical infrastructure networks [23], considering their ageing [15], [26]-[31], inside dependencies [1]-[2], [10] and outside impacts [4], [12]-[22] and the use of the achieved results to improve their safety [15], strengthen their resilience and mitigate [3] the effects of their degradation and failures.

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Krzysztof KOŁOWROCKI, Beata MAGRYTA-MUT

Gdynia Maritime University (Gdynia, Poland)

Safety and cost optimization of maritime transportation systems

To tie the investigations of the complex technical system safety together with the investigations of its operation the semi-Markov process model [1] can be used to describe this system operation process [2]–[3]. This model, under the assumption on the system safety structure multistate model [4]–[5], can be used to construct the general safety model of the complex multistate system changing its safety structure and its components safety parameters during variable operation process [2]–[3]. Further, using this general model, it is possible to find the complex system main safety characteristics such as the system safety function, and the system mean lifetimes in system safety subsets [2]. Having the system mean lifetimes in the system safety subsets and the system conditional instantaneous operation costs in the safety state subsets, it is possible to change the system operation process through applying the linear programming [6] for minimizing the system operation cost. The model for minimizing the mean value of the system operation cost is created and applied to the maritime ferry technical system.In this paper we defined optimal safety function as well as risk function for the maritime ferry technical system. Other optimal, practically significant critical infrastructure safety indicators specified also in the paper are its mean lifetime up to the exceeding a critical safety state, the moment when its risk function value exceeds the acceptable safety level, the critical infrastructure intensity of ageing/degradation, the coefficient of operation process impact on critical infrastructure intensities of ageing and the coefficient of critical infrastructure resilience to operation process impact [7]–[13]. Having the system operation process characteristics and particularly the conditional total mean values at the particular operation states over the fixed time of the system operation it is possible to alter the system operation process through applying the linear programming [6] in order to maximize the ferry technical system mean lifetime in the safety state subset not worse than the critical safety state. In the paper, the model for finding and maximizing the mean value of the system lifetime in the safety state subset not worse than the critical safety state is created and applied also to optimization of the maritime ferry technical system operation process. The results are compared with the values of the maritime ferry technical system safety indicators and costs before the optimization. The procedure of using the general safety analytical model of complex multistate technical system related to its operation process and the linear programing [14] is presented and applied to the optimization of the operation cost of the maritime ferry technical system. The mean value of the considered system total unconditional operation cost is evaluated and minimize through its operation process modification. The optimization procedure applied also to safety and resilience optimization of the ferry technical system influenced by its operation process gives practically important possibility of its safety improvement through its new operation strategy. The proposed optimization procedure can be used in operation and safety optimization of members of various real critical infrastructures [15]-[20]. Further research can be related to other impacts, for instance to climate-weather factors [21], and resolving the issues of critical infrastructure safety optimization and discovering optimal values of safety and resilience indicators of system impact by climate-weather conditions. These developments can also benefit the mitigation of critical infrastructure accident circumstances [22] and to improve critical infrastructure resilience to operation and climate-weather conditions [21].

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Jan KOROŃSKI

Politechnika Krakowska (Kraków, Poland)

Irena Łojczyk-Królikiewicz (1922-2019) w kontekście krakowskiej szkoły nierówności różniczkowych Jacka Szarskiego (1921-1980)

Irena Łojczyk-Królikiewicz urodziła się 9.08.1922 roku w Warszawie w rodzinie Tadeusza Królikiewicza (1888-1970) i Herminy z domu Wachtel (1892-1973). Rodzice Ireny Łojczyk-Królikiewicz pochodzili ze Lwowa. W latach 1931-35 ukończyła czteroklasową prywatną żeńską Szkołę im. Tymińskiej w Warszawie, a w latach 1935-38 żeńskie czteroklasowe Gimnazjum im. Juliusza Słowackiego w Warszawie. W roku szkolnym 1938-39 rozpoczęła naukę w prywatnym żeńskim Liceum Architektury w Warszawie. Ten etap kształcenia przerwał wybuch II wojny światowej. Irena Królikiewicz wraz z rodziną przeniosła się do Mielca. Tam w latach 1939-1942 w ramach tajnego nauczania ukończyła Liceum Matematyczno-Fizyczne uzyskując maturę w 1942 r. Po maturze, w latach 1942-1945, pracowała w zakładzie przemysłu drzewnego, prowadzonego przez jej rodziców, którzy w Mielcu posiadali ten zakład, wytwarzający początkowo meble a potem stolarkę budowlaną. Zakład ten ojciec Ireny Królikiewicz zakupił na korzystnych warunkach jako były legionista. W latach 1945-1949 Irena Królikiewicz studiowała na Wydziale Filozoficznym UJ z głównym kierunkiem matematyka. W 1949 roku uzyskała magisterium; promotorem był prof. Franciszek Leja. W 1946 podjęła dodatkowo studia na Wydziale Architektury PK. Od 1 września 1949 do 30 września 1951 r. pracowała na etacie młodszego asystenta w Katedrze Geometrii Wykreślnej, którą wtedy kierował prof. Antoni Plamitzer (1889-1954). Następnie była zatrudniona na etacie asystenta w Katedrze Matematyki (na Wydziale Lądowo-Wodnym PK) kierowanej przez zastępcę profesora Zdzisława Siedmiograja (1906-1962). Zaś od 1 kwietnia od 1955 do 1970 r. pracowała jako adiunkt w Katedrze Matematyki PK. 8 grudnia 1960 r. uzyskała doktorat na Uniwersytecie Jagiellońskim pod kierunkiem prof. Mirosława Krzyżańskiego (1907-1965). 4 marca 1968 r., już kilka lat po śmierci jej mistrza naukowego prof. Krzyżańskiego, uzyskała habilitację w Instytucie Matematyki Politechniki Warszawskiej. W konsekwencji habilitacji od 1 czerwca 1970 do 28 lutego 1991 r. pracowała na etacie docenta w Instytucie Matematyki PK. 1 marca 1991 r. została mianowana profesorem PK. Od 1 września 1988 do emerytury (30 września 1994 r.) sprawowała funkcję kierownika Zakładu Analizy Matematycznej w IM PK. Opublikowała około 30 istotnych prac naukowych z teorii równań i nierówności różniczkowych zwyczajnych i cząstkowych. Jej twórczość naukowa początkowo dotyczyła równań różniczkowych typu parabolicznego i powstawała pod kierunkiem jej promotora prof. Mirosława Krzyżańskiego. Z biegiem lat, po nagłej i przedwczesnej śmierci Krzyżańskiego w 1965 roku, nawiązała również współpracę z prof. Jackiem Szarskim - twórcą krakowskiej szkoły nierówności różniczkowych (powstałej w ramach słynnej krakowskiej szkoły równań różniczkowych Tadeusza Ważewskiego (1896-1972)), co wpłynęło na poszerzenie jej zainteresowań naukowych o teorię nierówności różniczkowych w zastosowaniu do układów równań różniczkowo-funkcjonalnych typu eliptycznego i parabolicznego, głównie nieliniowych. W tej dziedzinie uzyskła znaczące w skali świata nowe wyniki naukowe. W rozważanych przez siebie układach równań zaprzestała wykorzystywania warunku quasi-monotoniczności, który był definiowany i stosowany przez innych autorów. Irena Łojczyk-Królikiewicz wypracowała nowe podejście do rozważanych zagadnień naukowych zarówno dla równań zwyczajnych, jak i cząstkowych. Na szczególną uwagę zasługuje wprowadzona przez nią metoda funkcji stosownych (appropriate functions). Nowość tego podejścia polega na zdefiniowaniu klasy tzw. funkcji stosownych, które przy pewnym dodatkowym założeniu (separatywności) stają się funkcjami górnymi i dolnymi dla rozważanego zagadnienia.

Jest to warunek zarówno konieczny, jak i wystarczający do tego, by zbiory funkcji dolnych oraz górnych były rozdzielone. Wynika stąd możliwość efektywnej konstrukcji dwóch ciągów monotonicznych jednostajnie zbieżnych, wyznaczających w granicy jednoznaczne rozwiązanie rozważanych problemów. Będąc już na emeryturze nadal pracowała naukowo. W tym czasie napisała dwie niewielkie monografie dotyczące nierówności różniczkowych. Zmarła 22.07.2019 roku i została pochowana na Cmentarzu Komunalnym w Wieliczce.

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Bogumiła KOWALCZYK, Adam LECKO

University of Warmia and Mazury in Olsztyn (Olsztyn, Poland)

On Hankel determinants in subclasses of univalent functions

Some results on the third Hankel determinant $H_{3,1}(f)$ of functions f in selected subclasses of univalent functions are presented. Particularly, the upper bound of $|H_{3,1}(f)|$ in the class of functions of bounded boundary rotation is discussed.

Mateusz KRUKOWSKI

Instytut Matematyki Politechniki Łódzkiej (Lódź, Poland)

Kilka słów o programujących matematykach

Podczas mojego referatu podzielę się z Państwem moim kilkuletnim doświadczeniem z zajęć programistycznych dla matematyków. W prezentacji postawię pytanie o "najlepszy" język programowania dla celów matematycznych i dokonam analizy możliwych odpowiedzi na to pytanie. Mimo braku jednoznacznej konkluzji ("idealny język programistyczny" prawdopodobnie nie istnieje) postaram się wskazać te cechy, które maksymalnie ułatwiają studentom przyswojenie podstaw programowania. Zapraszam serdecznie!

Sushil KUMAR

Bharati Vidyapeeth's College of Engineering (Delhi–110063, India)
e-mail: sushilkumar16n@gmail.com

Differential subordination for certain Carathéodory functions

In this note, we consider certain Carathéodory functions defined on the open unit disk which are closely associated with the bounded regions in the right half of the complex plane. Several inclusions between such Carathéodory functions and starlike functions are addressed. The key tools in the proof of these inclusions are the theory of first order differential subordination and the admissibility condition technique. As application, these inclusions immediately yield sufficient conditions for analytic functions to be in various well-known subclasses of starlike functions.

AMS Subject Classification: Primary 30C45, 30C50.

Joint work with Prof. V. Ravichandran, National Institute of Technology, Tiruchirappalli, India.

Mirosław LACHOWICZ

University of Warsaw, Faculty of Mathematics, Informatics and Mechanics
(Warsaw, Poland)

Integro–differential *kinetic* equations and this strange world

I am going to present a theory of a class of nonlinear integro–differential equations — see ([5, 6, 7]). The equations may be related to *mesoscopic* (kinetic) level of description.

Moreover linear integro–differential equations corresponding to Markov processes at the *microscopic* level may be considered — cf. [1].

I will discuss the applications in

- Social Sciences: opinion formation — see [8, 9], redistribution inside a domain — see [3]),
- Economic science ([10]),
- Biology ([2]),
- Medicine ([4]).

I am going to relate self–organization processes with the possibility of blow-ups of solutions of the nonlinear equations.

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See Keong LEE

Universiti Sains Malaysia (Penang, Malaysia)

Zalcman conjecture

In geometric function theory, one of the famous theorems is the Bieberbach conjecture. For an analytic function $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ which is univalent on $\mathbf{D} = \{z \in \mathbf{C} : |z| < 1\}$, Bieberbach [1] in 1916 conjectured that $|a_n| \leq n$ for $n \in \mathbf{N}$. Since then, many works have been done to prove this conjecture, and it is not until 1985 that de Branges solved it completely [2]. During the 70 years, as an effort to solve the conjecture, many other conjectures have appeared, of which their validity would imply the Bieberbach conjecture. Particularly, de Branges proved the Milin conjecture to prove the de Branges conjecture. With the Bieberbach conjecture solved, many of these conjectures are solved as well. However, there are some which are still an open problem. The Zalcman conjecture is one of them. In this talk, we will revisit this conjecture and discuss about its development.

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Adam LIGĘZA

University of Warsaw (Warsaw, Poland)

Hamiltonians of Painlevé IV equation

In this talk I will speak about Painlevé equations, especially about the fourth Painlevé equation P_{IV}. I will discuss three different Hamiltonians and Hamiltonian systems connected with P_{IV} (that are Okamoto's Hamiltonian, rational Hamiltonian and Kecker's Hamiltonian) and a method how they can be matched with usage of algebraic geometry tools. I will show how that can be done by matching surface roots on the level of Picard lattice. Moreover I will check whether our matching is canonical.

This is a joint work with Galina Filipuk, Anton Dzhamay and Alexander Stokes.

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Abdallah K. LYZZAIK

Department of Mathematics, American University of Beirut (Beirut, Lebanon)

Valency Crietria for Harmonic Mappings of Bounded Boundary Rotation

The purpose of this talk is to investigate the valency of some classes of harmonic mappings of bounded boundary rotation.

The results that will be presented relate to the recent works [D. Bshouty and A. Lyzzaik, *Close-to-convexity criteria for planar harmonic mappings*, Complex Analysis Oper. Theory **5** (2011), 767–774], [D. Bshouty, A. Lyzzaik, and M. Sakar, *Harmonic Mappings of Bounded Boundary Rotation*, Proc. Amer. Math. Soc. **146** (2018), 1113–1121], [D. Bshouty and A. Lyzzaik, *A valency criteria for harmonic mappings*, CMFT **19** (2019), Issue 3, 433–454], and [T. Hayami, *A sufficient condition for $p-$valency of harmonic mappings*, Complex Var. Elliptic Equ. **59** (2014), 1214–1222].

Somya MALIK

National Institute of Technology (Tiruchirappalli, India)

Radius of Starlikeness for Bloch Functions

An analytic function f defined on the unit disk $\mathbf{D}$ belongs to the class $\mathcal{B}$ of Bloch functions if

$$f(0) = 0, \ f^{'}(0) = 1, \ \sup_{z\in\mathbf{D}} |f^{'}(z)|(1 - |z^2|) \leq 1.$$

We estimate the radii for $f \in \mathcal{B}$ to be a member of various known classes of analytic functions such as the class of parabloic starlikeness, subclasses of starlike functions associated with exponential function, cardioid, lune and a rational function. The results obtained are shown to be sharp.

The talk is based on joint work with Prof. V. Ravichandran.

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Jacek MARCHWICKI

University of Warmia and Mazury in Olsztyn (Olsztyn, Poland)

Various faces of achievement sets

The purpose of the poster is to introduce readers with achievement set, that is the set of subsums of the series $\sum_{n=1}^{\infty} x_n$, more precisely

$$A(x_n) = \left\{ \sum_{n=1}^{\infty} \varepsilon_n x_n : (\varepsilon_n) \in \{0,1\}^{\mathbf{N}} \right\} = \left\{ \sum_{n \in A} x_n : A \subset \mathbf{N} \right\}.$$

The poster includes some basic facts, theorems and examples related to the topic and four different faces of achievement sets:

(1) Kakeya conditions, that is how the inequalities between terms x_n and tails $r_n = \sum_{k=n+1}^{\infty} x_n$ affect the form of the set $A(x_n)$;
(2) Cardinal functions, when we count for how many different ways the elements of $A(x_n)$ are obtained;
(3) Recovering the sequence (x_n), when the achievement set is known;
(4) Aspects of conditionally convergent series on the plane.

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Ivan MATYCHYN[1], Viktoriia ONYSHCHENKO[2]

[1] *Uniwersytet Warmińsko-Mazurski* (Olsztyn, Poland)
[2] *National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute"* (Kijów, Ukraine)

On optimal control of linear fractional differential equations with variable coefficients

Fractional differential equations (FDEs) provide a powerful tool to describe memory effect and hereditary properties of various materials and processes [1, 2]. While linear systems of FDEs represent a fairly well investigated field of research, relatively few papers deal with linear FDEs involving variable coefficients. Meanwhile, models of many real-life systems and processes are described in terms of linear FDEs with variable coefficients, e.g. linearized aircraft models, linearized models of population restricted growth, models related to the distribution of parameters in the charge transfer and the diffusion of the batteries etc. Explicit solutions to linear systems of differential equations provide basis to perform stability analysis and to solve control problems. Analytical solutions of the linear systems of fractional differential equations with constant coefficients were derived in the papers [3, 4, 5] and then applied to solving optimal control problems in [6, 7, 8, 9]. Explicit solutions to linear systems of differential equations are usually expressed in terms of state transition matrix. In the case of FDEs with constant coefficients the state transition matrix can be represented using the matrix Mittag-Leffler function [3, 4].

Only a few papers are devoted to solutions of the systems of FDEs with variable coefficients and their control. Solution to the initial value problem for a linear system with variable coefficients involving Caputo derivative was obtained in [10]. In [11] explicit solutions for the linear systems of initialized [12] FDEs are obtained in terms of generalized Peano–Baker series [13]. Linear systems of FDEs with variable coefficients and their state-transition matrices are also discussed in [14, 15].

This paper deals with the initial value problem for linear systems of FDEs with variable coefficients involving Riemann–Liouville and Caputo derivatives. For these systems solution of initial-value problem is derived in terms of the generalized Peano–Baker series and time-optimal control problem is formulated. The optimal control problem is treated from convex-analytical viewpoint. Necessary and sufficient conditions for time-optimal control similar to that of Pontryagin's maximum principle are obtained. Theoretical results are supported by an example.

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Piotr MICHALAK

Gdynia Maritime University (Gdynia, Poland)

Minimal generating set of directed unoriented Reidemeister moves

Basic way of knot (link) representation is via knot (link) diagrams. Kurt Reidemeister (1927), James Alexander and Garland Baird (1926) independently showed that two diagrams of same knot can be related, up to planar isotopy, by set of three moves, today knowed as Reidemeister Moves. Michael Polyak in [1] shows analogous sets of moves for oriented link diagrams and shows which such set is minimal (has the least amount of moves). Piotr Suwara in [2] extends this result to minimal generating set of directed, oriented Reidemeister moves. In this talk minimal generating set of directed, unoriented Reidemeister Moves is shown. Ommiting orientation of diagrams yields possibility to reduce set of moves. First set of directed, unoriented Reidemeister Moves is presented, then it is shown that it can be reduced using planar isotopies. Later some of the moves are shown to be generated by other moves from the set. To obtain minimal generating set knot (link) invariants are used to show which moves are compulsory for set to be generating. Minimality of such sets is usefull for various proofs in field of a knot theory.

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Maria T. NOWAK

Maria Curie-Skłodowska University (Lublin, Poland)

On kernels of Toeplitz operators

Let H^2 denote the standard Hardy space on the unit disk $\mathbf{D}$ and let $\mathbf{T} = \partial\mathbf{D}$. For $\varphi \in L^\infty(\mathbf{T})$ the Toeplitz operator on H^2 is given by $T_\varphi f = P_+(\varphi f)$, where P_+ is the orthogonal projection of $L^2(\mathbf{T})$ onto H^2. It is known that the kernel of a T_φ is a subspace of H^2 of the form $\operatorname{Ker} T_\varphi = fK_I$, where $K_I = H^2 \ominus IH^2$ is the model space corresponding to the inner function I such that $I(0) = 0$ and f is an outer function of unit H^2 norm that acts as an isometric multiplier from K_I onto fK_I. Moreover, f can be expressed as

$$f = \frac{a}{1 - Ib_0}, \tag{1}$$

where a and b are functions from the unit ball of H^∞ such that $|a|^2 + |b_0|^2 = 1$ a.e. on $\mathbf{T}$, the pair (b_0, a) is special and $\left(\frac{a}{1-b_0}\right)^2$ is a rigid function in H^1. Then we also have $T_{\bar{I}\bar{f}/f} = fK_I$

In the recent paper [1], the authors considered the Toeplitz operator $T_{\frac{\bar{g}}{g}}$ where $g \in H^\infty$ is outer. Among other results, they described all outer functions g such that $\operatorname{Ker} T_{\frac{\bar{g}}{g}} = K_I$. In the talk we describe all such functions g for which $\operatorname{Ker} T_{\frac{\bar{g}}{g}} = fK_I$. We also discuss properties of the kernels of Toeplitz $T_{\bar{I}\bar{f}/f}$, where f is an outer function in H^2 given by (1), I is inner such that $I(0) = 0$, but (b_0, a) is not special or the function $f_0^2 = \left(\frac{a}{1-b_0}\right)^2$ is not rigid.

The talk is based on joint work with P. Sobolewski, A. Sołtysiak and M. Wołoszkiewicz-Cyll.

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Krzysztof PANCERZ

State School of Higher Education in Zamość (Zamość, Poland)

Classification Strategies in Classification and Prediction Software System (CLAPSS)

Classification and Prediction Software System (CLAPSS) [1] is a tool developed for solving different classification and prediction problems using, among others, some specialized approaches based mainly on fuzzy sets, rough sets as well as decision trees. The tool is equipped with a userfriendly graphical interface. We present a new functionality, recently added to the tool, concerning non-standard classification strategies. The multi-class decision problems can be solved using the following strategies: one against all, one against one, hierarchically-dichotomous. The underlying classification is performed using the selected decision tree and rule based algorithms implemented in the WEKA system [2].

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Maciej PAROL

The John Paul II Catholic University of Lublin (Lublin, Poland)

Kaplan classes for polynomials with all zeros on unit circle

The presented results concern polynomials of the form

$$\mathbf{D} \ni z \mapsto P_n(z;T_n) := \prod_{k=1}^{n} \left(1 - z\mathrm{e}^{-\mathrm{i}t_k}\right) \ ,$$

where $\mathbf{N} \ni n \mapsto T_n := (t_1, t_2, \ldots, t_n)$ is an increasing sequence of values from $[0; 2\pi)$ such that $t_1 := 0$. Complete membership to Kaplan classes of the polynomial $P_n(z;T_n)$ was proved in [1], expanding the result of Jahangiri [3]. This result was generalized in [2], by raising factors $1 - z\mathrm{e}^{-\mathrm{i}t_k}$ of a polynomial $P_n(\cdot;T_n)$ to positive real powers.

The presentation is based on the results obtained together with Szymon Ignaciuk.

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Dariusz PARTYKA[1], Ken-ichi SAKAN[2]

[1]*The John Paul II Catholic University of Lublin* (Lublin, Poland),
The State School of Higher Education in Chełm (Chełm, Poland)
[2]*Graduate School of Science, Osaka City University* (Osaka, Japan)

A simple deformation of harmonic mappings

Every complex valued harmonic function F in the unit disk $\mathbf{D} := \{z \in \mathbf{C} : |z| < 1\}$ has the unique decomposition $F = H + \overline{G}$, where H and G are holomorphic functions in $\mathbf{D}$ and $G(0) = 0$. Therefore, the following *simple deformation* of a given harmonic function $F : \mathbf{D} \to \mathbf{C}$,

$$\mathbf{D} \ni z \mapsto F_t(z) := H(z) + t\overline{G(z)}, \quad t \in \mathbf{C},$$

can be defined. The talk is a survey of various results involving the injectivity, quasiconformality and close-to-convexity of F_t under certain assumptions on F and t. The bi-Lipschitz property of $F_t \circ H^{-1}$ will be also considered provided H is injective. In particular, the case will be discussed where F is a sense-preserving harmonic mapping in $\mathbf{D}$, its holomorphic part H is injective in $\mathbf{D}$ and $H(\mathbf{D})$ is a rectifiably M-arcwise connected domain for a certain $M \geq 1$, i.e., for all $z, w \in H(\mathbf{D})$ there exists an arc γ joining the points z and w in $H(\mathbf{D})$ with the length $|\gamma|_1 \leq M|w - z|$. Then for all $z, w \in \mathbf{D}$,

$$(1 - M|t|\|\mu_F\|_\infty)|H(z) - H(w)| \leq |F_t(z) - F_t(w)| \leq (1 + M|t|\|\mu_F\|_\infty)|H(z) - H(w)|,$$

and so $F_t \circ H^{-1}$ is bi-Lipschitz provided $M|t|\|\mu_F\|_\infty < 1$. Moreover, the following implications hold:

(i) If $M|t|\|\mu_F\|_\infty \leq 1$ and G'/H' is not a constant function, then F_t is an injective mapping, where μ_F is the complex dilatation of F.

(ii) If $M|t|\|\mu_F\|_\infty < 1$, then F_t is a K-quasiconformal mapping with $K \leq \frac{1+|t|\|\mu_F\|_\infty}{1-|t|\|\mu_F\|_\infty}$.

Note that $H(\mathbf{D})$ is a convex domain if and only if $H(\mathbf{D})$ is a rectifiably 1-arcwise connected domain. Thus both the implications are valid with $M := 1$, provided $H(\mathbf{D})$ is a convex domain.

James F. PETERS

Computational Intelligence Laboratory, University of Manitoba
WPG, MB, R3T 5V6 (Canada)
Department of Mathematics, Faculty of Arts and Sciences, Adiyaman University
02040 Adiyaman (Turkey)

Amiable Fixed Sets.
Extension of the Brouwer Fixed Point Theorem

Dedicated to Zdzisław Pawlak and Andrzej Skowron

This paper introduces planar amiable fixed point sets resulting from **continuous self-maps** (csm's) from $\mathbf{R}^2$ to itself and proximal planar fixed sets that result from **descriptive proximally continuous self-maps** (dpc's) from a descriptive proximity space to itself. A **descriptive proximity** $\boldsymbol{\delta_\Phi}$ is a relation between the descriptions of a pair of sets. Each **description** is a feature vector that profiles a nonempty set. We write $\boldsymbol{A\ \delta_\Phi\ B}$, provided sets A and B have matching descriptions. A nonempty set X equipped with δ_Φ is a **descriptive proximity space**.

A $f : (X, \delta_\Phi) \to (X, \delta_\Phi)$ is a **descriptive proximally continuous map**, provided there is a least one pair of nonempty sets $A, B \subset X$ such that $A \ \delta_\Phi \ B$ implies that $f(A) \ \delta_\Phi \ f(B)$. That is, a dpc preserves the descriptive proximity between sets. A dpc map is an extension of an Efremovič-Smirnov proximally continuous (pc) map introduced during the early-1950s by V.A. Efremovič and Yu. M. Smirnov. A basic result in this work is that every dpc has a proximal fixed point set. This result stems from an extension of the Brouwer Fixed Point Theorem (BFPT). A pair of sets A and $f(A)$ (for dpc f) are **amiable fixed sets**, provided $f(A) \ \delta_\Phi \ A$.

Yet another extension of the BFPT stems from dpc's in a cyclic group presentation of a **cycle** that is a collection of path-connected vertexes, providing a rich source of fixed point sets. A **free group presentation of a cycle** is a self map from a nonvoid cycle to a corresponding free finitely-generated group. A basic result is that the contour of every shape complex in a Whitehead closure-finite weak (CW) space has a free cyclic group presentation. Descriptive fixed sets and the Betti numbers that quantify free Abelian group presentations of shapes are derived from dpc's relative to the description of the boundary regions of the shapes. **Approximate descriptive fixed sets** are derived from dpc's by relaxing the matching description requirement for the descriptive closeness of the fixed sets. This leads to a plethora of video processing applications in which the closeness of fixed video frame shapes is approximate rather than exact.

2010 Mathematics Subject Classification. 55M20 (fixed points in algebraic topology), 54E05 (proximity structures), 11E57 (cyclic groups).

Key words and phrases. Amiable, Approximate, Brouwer Fixed Point Theorem, Continuous Self Maps, CW Space, Cycle, Descriptive Fixed Set, Descriptive Proximally Continuous Map, Descriptive Proximity, Fixed Point Set, Free Group Presentation, Path-Connected.

V. RAVICHANDRAN

National Institute of Technology (Tiruchirappalli, India)

A Survey on Univalent Functions with Fixed Second Coefficient

The bound on the second coefficient of normalized univalent functions defined in the open unit disk leads to covering, distortion, growth and other results. In view of the importance, the functions with fixed second coefficient were widely studied by geometric function theorists for over a century and has a vast literature. The study was initiated by T. H. Gronwall (1916-1920). The extended form of the Classical Schwarz Lemma proved by M. Finkelstein (1967) is the main foundation result in this theory. We provide a survey of the various results for these functions when they have further geometric properties such as starlikeness, convexity or close-to-convexity. We shall focus on the classical growth and distortions results, radius problems and subordination theory for these functions.

The central idea behind computing the growth, distortion and radius estimates is to find the bounds for the quantities $|p(z)|$, $\operatorname{Re} p(z)$, $|p'(z)|$, $|zp'(z)/p(z)|$, $\operatorname{Re}(zp'(z)/p(z))$ and other associated functionals where p varies over the class of analytic functions with positive real part in the unit disk with $p(0) = 1$ and fixed initial coefficient $p'(0)$. This technique was employed by several authors like K. S. Padmanabhan (1969), D. E. Tepper (1970), H. S. Al-Amiri (1971), C. P. McCarty (1972-74), H. Silverman (1972), O. P. Juneja & M. L. Mogra (1978-84) and P. D. Tuan and V. V. Anh (1980) to name a few.

In 2011, the classical theory of differential subordination by S. S. Miller and M. T. Mocanu (1981) was extended for functions with fixed initial coefficient. This modified theory establishes the influence of the second coefficient in certain differential implications associated with starlike and convex functions with fixed second coefficient including the well-known Marx Strohhäcker Theorem and Open Door Lemma.

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Swadesh Kumar SAHOO

Indian Institute of Technology Indore (Indore, India)

Bohr inequalities for certain integral operators

We present sharp Bohr-type radii for certain complex integral operators defined on a set of bounded analytic functions in the unit disk.

Asha SEBASTIAN

National Institute of Technology (Tiruchirappalli, India)

Radius of starlikeness of certain classes of analytic functions

The class $\mathcal{P}$ of analytic functions $p(z) = 1 + cz + \cdots$ on the open unit disk having positive real part is known as the class of Carathéodory functions or the class of functions with positive real part. We consider three classes of functions defined using the class $\mathcal{P}$ and study the radii properties. Let $\Phi(z) = 1/(1+z)$ be a function defined on the unit disc $\mathbf{D}$. The classes of normalised analytic functions f that we discuss are the following: (i) $f/g \in \mathcal{P}$ and $g/(z\Phi) \in \mathcal{P}$ (ii) $|(f(z)/g(z)) - 1| < 1$ and $g/(z\Phi) \in \mathcal{P}$ (iii) $f/(z\Phi) \in \mathcal{P}$, where g is some normalised analytic function. We shall discuss the radii of these functions to belong to various subclasses of starlike functions like starlike functions of order α, parabolic starlike functions, and the classes of starlike functions associated with lemniscate of Bernoulli, reverse lemniscate, sine function, a rational function, cardioid, lune. The radii obtained for these classes of functions are sharp.

The talk is based on joint work with Prof. V. Ravichandran.

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Kanika SHARMA

Atma Ram Sanatan Dharma College, University of Delhi (Delhi, India)

Radius of starlikeness for two classes of analytic functions

For the class of all normalized analytic functions f on the open unit disc for which there are normalized analytic functions g and p such that $f(z)/g(z)$, $g(z)/zp(z)$ and $p(z)$ are subordinate to $\sqrt{1+z}$, we have computed radii of starlikeness of order α, parabolic starlikeness, and other radii related to domains bounded by certain cardioid, and to the exponential function e^z. Similar results are investigated when $\sqrt{1+z}$ is replaced by the exponential function e^z.

This is the joint work with V. Ravichandran and Rosihan M. Ali

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Meghna SHARMA

University of Delhi (Delhi, India)

Higher Order Differential Subordination for the functions with Positive Real part using Admissibility Technique

Let $\mathcal{P}$ denote the class of functions with positive real part of the form $p(z) = 1 + c_1 z + c_2 z + \cdots$ over $\mathbf{D}$. We shall discuss various first, second and third order differential subordination relations associated with certain functions belonging to class $\mathcal{P}$ using the admissibility conditions. Sufficient conditions are obtained so that the function p, normalized by the condition $p(0) = 1$ is subordinate to Janowski function, whenever $\psi(p(z), zp'(z), z^2p''(z), z^3p'''(z); z)$ is subordinate to (i) Modified Sigmoid function, (ii) Exponential Function, and (iii) Janowski function, for some analytic function ψ.

This is a joint work with Naveen Kumar Jain and Sushil Kumar.

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Young Jae SIM[1] and Derek K. THOMAS[2]

[1]*Kyungsung University* (Busan, Korea)
[2]*Swansea University* (Swansea, United Kingdom)

An overview of results on Successive Coefficients of Univalent Functions

Let f be analytic in the unit disk $\mathbf{D} = \{z \in \mathbf{C} : |z| < 1\}$, and $\mathcal{S}$ be the subclass of normalized univalent functions given by $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ for $z \in \mathbf{D}$. We give a survey of the important results to date concerning the problem of finding upper and lower bounds for $|a_{n+1}| - |a_n|$ when $f \in \mathcal{S}$, and the more significant subclasses.

Let F be the inverse function of f, and be given by $F(\omega) = \omega + \sum_{n=2}^{\infty} A_n \omega^n$, valid on some disk $|\omega| \leq r_0(f)$. Similar problems for the upper and lower bounds of $|A_{n+1}| - |A_n|$ are considered.

Young Jae SIM[1] and Derek K. THOMAS[2]

[1]*Kyungsung University* (Busan, Korea)
[2]*Swansea University* (Swansea, United Kingdom)

Bounds for the fifth coefficients of analytic functions

Let $f \in \mathcal{A}$, the class of normalized analytic functions defined in the unit disk $\mathbf{D}$ and given by $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ for $z \in \mathbf{D}$. We discuss a new approach, based on results in [1, 2, 3], to finding bounds for some functionals involving the fifth coefficient a_5. Improved bounds are obtained for $|a_5|$ for the classes of strongly starlike, gamma starlike, and the $\mathcal{B}_1(\alpha)$ Bazilevič functions. A sharp bound for the fourth logarithmic coefficient for the class of gamma starlike functions is also obtained.

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Sanjeev SINGH

Indian Institute of Technology Indore (Indore, India)

Zeros and geometric properties of hyper-Bessel functions

The reality of zeros of special function plays an important role in the study of geometric properties of such special functions. In this talk, the real and complex zeros of some special entire functions such as Wright, hyper-Bessel, and a special case of generalized hypergeometric functions will be discussed by using some classical results of Laguerre, Obreschkhoff, Polya, and Runckel. Moreover, some geometric properties of normalized hyper-Bessel functions will be presented. Especially we focus on the radii of starlikeness, convexity, and uniform convexity of hyper-Bessel functions and we show that the obtained radii satisfy some transcendental equations.

Srikandan SIVASUBRAMANIAN

Department of Mathematics, University College of Engineering
Tindivanam Anna University (Chennai, India)

On a class of analytic functions related to Robertson's formula and subordination

In 1981, Robertson [10] pointed out although the class of starlike functions with respect to order α $(0 \leq \alpha < 1)$ has been explored extensively by many authors over a long period of time, not much seems to be known about the class of analytic functions G that map a domain D onto the open unit disc Δ that are starlike with respect to a boundary point. However, an extensive exploration is yet to be done on this concept. In the present talk, few classes of analytic functions in the unit disc by modification of the well known Robertson's analytic formula for starlike functions with respect to a boundary point combined with subordination are introduced and studied. An integral representation and growth theorem are proved. Early coefficients and the Fekete-Szegö functional are also estimated. The connections of the present work with those of the earlier concepts are pointed out.

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Bożena STARUCH, Bogdan STARUCH

University of Warmia and Mazury in Olsztyn (Olsztyn, Poland)

Parametrized family of similarity-based classifiers

We use a family of point classifiers to predict rare events such as seismic shocks in a mine. The point classifier analyzes sensor readings taken at short intervals of time, takes into account specific circumstances and, using similarity to previous events, determines whether a given phenomenon will occur within a certain time, for example 10 hours. Then the sequence of values of the point classifiers (time series) is used to determine the probability of occurrence of the event within the specified time. The practical application of the determined probability may consist in generating different levels of warnings about the possible danger of occurrence of an unwanted event.

Toshiyuki SUGAWA

Tohoku University (Sendai, Japan)

Coefficient estimates of the Riemann mapping functions

The celebrated Riemann mapping theorem asserts that for each simply connected proper subdomain G of the complex plane $\mathbf{C}$ and a given point $a \in G$, there is a unique conformal (equivalently, analytic) homeomorphism $g : G \to \mathbf{D}$ such that $g(a) = 0$, $g'(a) > 0$. Here, $\mathbf{D}$ denotes the unit disk $|z| < 1$ in the complex plane. Such a function g is often called the Riemann mapping function of G (with basepoint a). By transforming G by the affine map $w \mapsto g'(a)(w-a)$ if necessary, we may assume that $a = 0$ and $g'(0) = 1$. Then we have a series expansion of the form

$$g(w) = w + b_2 w^2 + b_3 w^3 + \cdots$$

near the origin. Löwner [1] proved the sharp inequalities

$$|b_n| \le \frac{(2n)!}{n!(n+1)!} = \frac{1}{n+1}\binom{2n}{n}, \quad n = 2, 3, \ldots.$$

If we restrict the class of domains G, we should have better bounds. In this talk, a survey of the results in this direction will be given and recent progress will also be mentioned.

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Marek SVETLIK

University of Belgrade, Faculty of Mathematics (Belgrade, Republic of Serbia)

Some versions of the Schwarz lemma for harmonic mappings

In this talk, we present several generalizations of the Schwarz lemma for harmonic mappings. We consider harmonic mapping $f : \mathbf{U} \to \mathbf{U}$, where $\mathbf{U} = \{z \in \mathbf{C} : |z| < 1\}$. The version of the Schwarz lemma for such mappings are known in the case $f(0) = 0$. Using a novelty, we obtain appropriate results if $z = 0$ is not mapped to itself. Further, we give appropriate versions of the Schwarz lemma for harmonic mappings, whereby values of such mappings and the norms of their differentials at the point $z = 0$ are given. These results can be viewed as analogies of the corresponding theorems for holomorphic mappings. Note that the hyperbolic metric on the planar domains plays a crucial role in our investigations.

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Anbhu SWAMINATHAN

Indian Institute of Technology Roorkee (Roorkee, India)

Geometric Properties of Analytic Functions Associated with Nephroid Domain

In this talk, the Carathéodory function $\varphi_{Ne}(z) = 1 + z - z^3/3$ which maps the unit circle $\{z : |z| = 1\}$ onto a 2-cusped curve called nephroid given by $\left((u-1)^2 + v^2 - \dfrac{4}{9}\right)^3 - \dfrac{4v^2}{3} = 0$, and the function class $\mathcal{S}^*_{Ne}$ defined as

$$\mathcal{S}^*_{Ne} := \left\{ f \in \mathcal{A} : \frac{zf'(z)}{f(z)} \prec \varphi_{Ne}(z) \right\},$$

where $\prec$ denotes subordination are considered. Apart from discussing the characteristic properties of the region bounded by this nephroid, the structural formulas, extremal functions, growth and distortion results, inclusion results, coefficient bounds and Fekete-Szegö problems are discussed for the classes $\mathcal{S}^*_{Ne}$ and $\mathcal{C}_{Ne}$. Further, sharp estimates on $\beta \in \mathbf{R}$ so that the first-order differential subordination

$$1 + \beta \frac{zp'(z)}{p^j(z)} \prec \mathcal{P}(z), \quad j = 0, 1, 2,$$

implies $p \prec \varphi_{Ne}$, where $\mathcal{P}$ is certain Carathéodory function with nice geometrical properties and p is analytic satisfying $p(0) = 1$ are explained. As applications, sufficient conditions for $f \in \mathcal{A}$ to be in the class $\mathcal{S}^*_{Ne}$ are established. In continuation, sharp $\mathcal{S}^*_{Ne}$-radii for several geometrically defined function classes introduced in the recent past are outlined. Moreover, radii problems related to the families defined in terms of ratio of functions are also exhibited with graphical illustrations for all the possible situations. Results from other related literature are also provided. Problems for further research are highlighted at the end.

The talk is based on joint work with Lateef Ahmad Wani.

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Anna SZPILA

Uniwersytet Rzeszowski (Rzeszów, Poland)

Zdalne nauczanie w Uniwersytecie Rzeszowskim na kierunku matematyka – przebieg, problemy, efekty

Spowodowane pandemią przejście z dnia na dzień do całkowitego zdalnego nauczania postawiło przed całą społecznością akademicką wyzwanie – w jaki sposób poprowadzić kształcenie, aby uzyskane metodami na odległość efekty uczenia się były zgodne z efektami zakładanymi w programach studiów.

W referacie przedstawię drogę, którą przeszli nauczyciele akademiccy prowadzący zajęcia na kierunku matematyka w Uniwersytecie Rzeszowskim od tradycyjnego nauczania poprzez komunikację ze studentami za pomocą e-maili i udostępnianie wersji cyfrowej materiałów w celu samodzielnego zapoznania oraz zadań do samodzielnego rozwiązania do zajęć interaktywnych z użyciem narzędzia MS Teams i czy wszystkim się to udało. Zaprezentuję głos studentów kierunku matematyka oceniających w anonimowych ankietach nauczanie na odległość. Porównam strukturę ocen z egzaminów w sesjach prowadzonych kontaktowo i zdalnie.

Wszystkie zaprezentowane informacje powinny pomóc w odpowiedzi na pytania: *Czy zdalnie można wykształcić matematyka?* oraz *Co ze zdalnego nauczania można wykorzystać w czasie popandemicznym?*

Barbara Śmiarowska

University of Warmia and Mazury in Olsztyn (Olsztyn, Poland)

The fourth-order Hermitian Toeplitz determinant for convex functions

Given $q, n \in \mathbf{N}$, the Hermitian Toeplitz matrix $T_{q,n}(f)$ of $f \in \mathcal{A}$ of the form

$$f(z) = \sum_{n=1}^{\infty} a_n z^n, \quad a_1 := 1, \ z \in \mathbf{D} := \{z \in \mathbf{C} : |z| < 1\},$$

is defined by

$$T_{q,n}(f) := \begin{bmatrix} a_n & a_{n+1} & \dots & a_{n+q-1} \\ \overline{a}_{n+1} & a_n & \dots & a_{n+q-2} \\ \vdots & \vdots & \vdots & \vdots \\ \overline{a}_{n+q-1} & \overline{a}_{n+q-2} & \dots & a_n \end{bmatrix},$$

where $\overline{a}_k := \overline{a_k}$. Let $|T_{q,n}(f)|$ denote the determinant of $T_{q,n}(f)$.

Let $\mathcal{S}^c$ denote the class of convex functions, that is, univalent functions $f \in \mathcal{A}$ such that $f(\mathbf{D})$ is a convex domain in $\mathbf{C}$. It is well know that a function $f \in \mathcal{A}$ is in $\mathcal{S}^c$ if and only if

$$\operatorname{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 0, \quad z \in \mathbf{D}.$$

In recent years a lot of papers has been devoted to the estimation of determinants built with using coefficients of functions in the class $\mathcal{A}$ or its subclasses. Hankel matrices i.e., square matrices which have constant entries along the reverse diagonal (see e.g., [3] with further references), and the symmetric Toeplitz determinant (see [1]) are of particular interest.

In [4] the conjecture that the sharp inequalities $0 \le |T_{q,1}(f)| \le 1$ for all $q \ge 2$, holds over the class $\mathcal{S}^c$ was proposed and was confirmed for $q = 2$ and $q = 3$. Our purpose is to prove this conjecture for $q = 4$.

This presentation is based on joint work with A. Lecko and Y. J. Sim.

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SŁAWOMIR K. TADEJA, DIANA JANIK, PRZEMYSŁAW STACHURA

Immersive Toolbox Sp. z o. o. (Wólczańska 125, Łódź, Polska)

Object Assembly Assisted with Augmented Reality Interface and QR-Based Tagging: An Early Stage Report

Augmented Reality (AR) is one of the most promising branches of the so-called immersive interfaces. It allows for the digital content to be superimposed on the real view of the user presented on a screen of a mobile device, or with the help of specialized AR goggles. In the case of the former, typically the view captured by the front camera of a smartphone or a tablet is streamed on the device screen with superimposed digital artifacts, whereas goggles present the digital content on top of semitransparent glasses. Usually, by such content, we understood 3D shapes or models that the AR user can perceive as "real" objects immersed in the same 3D space surrounding the user. AR technology has a wealth of potential applications in both the industry and academia alike. These include, for instance, complex engineering tasks such as the design engineering or manufacturing and assembly processes. Furthermore, more and more use cases are being currently recognized where such technology can be successfully utilized to bring the most benefits to all the stakeholders. However, due to its novelty, there is still a range of obstacles that have to be overcome in order the fulfill the promise bestowed by AR technology. One such issue is the selection of the most expedient way in which the user can interact with real objects with the help of AR. To this end, we will present a system the combines the immersive interface offered by the state-of-the-art AR goggles coupled with QR-based tagging. The QR codes, developed for the purpose of fast and reliable readout in the industrial setting, seems to be a promising tool for facilitating interaction between the AR system, its user, and the real objects. Here, we will present an early stage report from the work concerning the use of AR goggles and QR codes in the simplified assembly task for teaching the trainee how to put together a locker.

MATEUSZ TORBICKI

Gdynia Maritime University (Gdynia, Poland)

Modelling critical infrastructure safety impacted by climate-weather change process

The paper is devoted to modeling and evaluating the critical infrastructure safety affected by extreme weather conditions using theorems related to the complex systems safety and semi-Markov processes. The changing weather conditions create the climate-weather change process (considered as the semi-Markov process) influencing on the critical infrastructure assets and its safety. The climate-weather change process parameters are defined and its characteristics are presented. The indicators reflecting the impact of the climate-weather change process on the critical infrastructure safety and resilience are proposed. Moreover, the safety and resilience analysis of the real critical infrastructure which is the port oil terminal critical infrastructure influenced by the climate-weather change process at its operating area is conducted on the basis of the introduced approach.

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Katarzyna TRĄBKA-WIĘCŁAW, Paweł ZAPRAWA

Lublin University of Technology (Lublin, Poland)

Estimates of coefficient functionals for functions convex in the imaginary-axis direction. Part I, II.

Let $\mathcal{A}$ be a class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

which are analytic in the open unit disk $\Delta = \{z \in \mathbf{C} : |z| < 1\}$. Let $\mathcal{S}^*$ denote the subclass of $\mathcal{A}$ consisting of starlike functions. Let $\mathcal{K}_{\mathbf{R}}(\mathrm{i})$ be the subclass of $\mathcal{A}$ consisting of functions convex in the direction of the imaginary axis and having all real coefficients.

Given that $\beta \in (-\pi/2, \pi/2)$ and $g \in \mathcal{S}^*$, a function $f \in \mathcal{A}$ is called close-to-convex with argument β with respect to g, if

$$\mathfrak{Re}\left\{\frac{\mathrm{e}^{\mathrm{i}\beta} z f'(z)}{g(z)}\right\} > 0, \quad z \in \Delta. \tag{1}$$

The class of all functions satisfying (1) is denoted by $\mathcal{C}_\beta(g)$.

In this paper we consider the subclass $\mathcal{C}_0(h)$ of close-to-convex functions $\mathcal{C}_\beta(g)$, where

$$h(z) = \frac{z}{1-z^2}, \quad z \in \Delta.$$

It follows from the definition of $\mathcal{C}_\beta(g)$ that

$$f \in \mathcal{C}_0(h) \Leftrightarrow \mathfrak{Re}\{(1-z^2) f'(z)\} > 0.$$

In this paper, we find bounds of the three following functionals

$$F_n = a_{n+1} - a_n, \quad G_n = (n+1)a_{n+1} - na_n, \quad S_n = 1 + a_2 + a_3 + \cdots + a_n$$

for functions from $\mathcal{C}_0(h)$. The bounds depend on the second coefficient of $f \in \mathcal{C}_0(h)$. For $f \in \mathcal{C}_0(h)$ we also derive bounds of the coefficient a_n providing that a_3 is fixed. The obtained results are sharp (see, [1]).

Remark. If $f \in \mathcal{C}_0(h)$, then f is convex in the direction of the imaginary axis. Moreover, if all coefficients of f are real, then

$$f \in \mathcal{C}_0(h) \Leftrightarrow f \in \mathcal{K}_{\mathbb{R}}(i).$$

For this reason, we can transfer the obtained results for $\mathcal{C}_0(h)$ onto the class $\mathcal{K}_{\mathbf{R}}$(i).

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Edyta TRYBUCKA[1], Renata DŁUGOSZ[2], Piotr LICZBERSKI[2]

[1]*Institute of Mathematics, University of Rzeszów* (Rzeszów, Poland)
[2]*Centre of Mathematics and Physics, Lodz University of Technology* (Łódź, Poland)
[3]*Institute of Mathematics, Lodz University of Technology* (Łódź, Poland)

On a new result for a family of even holomorphic functions of several complex variables

In the presentation we will consider some properties of a family K_G^- of holomorphic functions defined on bounded complete n-circular domain G of $\mathbf{C}^n$ by the evenness (see [3]). For instance, we remind some relationships between K_G^- and a few families, investigated by Bavrin (see [1]). We will present also some properties of this family, like as: the growth and distortion type theorems for f from K_G^- and the estimates of Minkowski G-balances of homogeneous polynomials which occur in power series expansion of f from K_G^-. The central point of the lecture makes a generalized version of the planar Fekete-Szegö coefficients problem for the family K_G^- (cf. [2]).

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Nikola TUNESKI[1], Milutin OBRADOVIĆ[2]

[1] *Ss. Cyril and Methodius University* (Skopje, Republic of North Macedonia)
[2] *University of Belgrade* (Belgrade, Serbia)

Coefficient inequalities for certain classes of univalent functions

In recent time, the problem of finding upper bound, preferably sharp, of the Hankel determinant, Zalcman conjecture and Generalised Zalcman conjecture for classes of univalent functions, is being rediscovered and attracts significant attention among the mathematicians working in the field. In that direction, this presentation will provide some new and improvements of existing results for the class of starlike functions and some of its subclasses. For the proofs we reveal some results of Prokhorov-Szynal ([1]) and Grunsky ([2]).

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Wojciech WALAT

Uniwersytet Rzeszowski (Rzeszów, Poland)

Główne problemy edukacji wyższej w związku z wprowadzeniem pełnego kształcenia zdalnego

Badania sondażowe przeprowadzone po całkowitym przejściu na kształcenie zdalne (w związku z pandemią COVID-19) pokazały, iż przed edukacją stoi prawdziwie trudne wyzwanie: jak od tradycyjnie rozumianej alfabetyzacji przejść do alfabetyzacji funkcjonalnej człowieka nowej generacji, tzw. homo interneticusa, który wierzy jedynie w moc swojego smartfona podłączonego do sieci, i jak dotąd pozostaje zadowolonym analfabetą funkcjonalnym[1]?

W artykule przedstawię trzy główne problemy związane z wirtualną edukacją wyższą, które są charakterystyczne dla osobowości homo interneticusa: utrata tożsamości studenta/nauczyciela akademickiego, akademicki efekt FOMO i rozproszenie poznawcze.

[1] Miliony Europejczyków czy Amerykanów mimo zakończenia edukacji formalnej pozostają analfabetami w ścisłym tego słowa znaczeniu lub analfabetami funkcjonalnymi – co oznacza, że ze zdobytej wiedzy nie potrafią zrobić racjonalnego użytku. Źródła tego stanu rzeczy upatruje się przede wszystkim w dominującej dziś komunikacji sieciowej, stąd określa się współczesnego człowieka mianem człowieka podłączonego do Internetu, czyli homo interneticus (internetus)

Lateef Ahmad WANI, Anbhu SWAMINATHAN

Department of Mathematics, Indian Institute of Technology, Roorkee (Uttarakhand, India)

Radius Constants For Functions Associated with a Limacon Domain

Let $\mathcal{A}$ be the collection of analytic functions f defined in $\mathbf{D}\,(|z| < 1)$ such that $f(0) = f'(0) - 1 = 0$. Using the concept of subordination ($\prec$), we define

$$\mathcal{S}^*_\ell := \left\{ f \in \mathcal{A} : \frac{zf'(z)}{f(z)} \prec \varphi_\ell(z) = 1 + \sqrt{2}z + \frac{z^2}{2},\ z \in \mathbf{D} \right\},$$

where the function $\varphi_l(z)$ maps $\mathbf{D}$ univalently onto the region Ω_ℓ bounded by the limacon curve

$$\left(9u^2 + 9v^2 - 18u + 5\right)^2 - 16\left(9u^2 + 9v^2 - 6u + 1\right) = 0.$$

Let $\mathbf{D}_r := \{z : |z| < r\}$ and $\mathcal{G}$ be some geometrically defined subclass of $\mathcal{A}$. In this paper, we find the largest number $\rho \in (0,1)$ and some function $f_0 \in \mathcal{G}$ such that for each $f \in \mathcal{G}$

$$\mathcal{Q}_f(\mathbf{D}_r) \subset \Omega_\ell \quad \text{for every} \quad r \leq \rho,$$

and

$$\mathcal{Q}_{f_0}(\partial\mathbf{D}_\rho) \cap \partial\Omega_\ell \neq \emptyset,$$

where the function $\mathcal{Q}_f : \mathbf{D} \to \mathbf{C}$ is given by

$$\mathcal{Q}_f(z) := \frac{zf'(z)}{f(z)}, \quad f \in \mathcal{A}.$$

Moreover, certain graphical illustrations are provided in support of the results proved in this paper.

Keywords: Subordination, Ma-Minda type classes, radius problems, lemniscate of Bernoulli, limacon

Maja WENDERLICH

Akademia Pedagogiki Specjalnej im. M. Grzegorzewskiej (Warszawa, Poland)

Doświadczenia krystalizujące w rozwijaniu uzdolnień matematycznych polskich laureatów olimpiad międzynarodowych IMO

W swoim referacie przedstawię doświadczenia krystalizujące mające związek z kształtowaniem się zainteresowań matematyką: to znaczy takie doświadczenia, które "angażują w znaczące i niezapomniane spotkanie osoby o niezwykłym talencie lub potencjalnych zdolnościach z tworzywem danego pola, w którym talent ten może się ujawnić" (Walters, Gardner, 1986). Badaniem zostali objęci polscy laureaci międzynarodowych olimpiad matematycznych organizowanych w latach 2000-2019. Z historii matematyki wynika, że przełomowych odkryć matematycznych dokonywali ludzie młodzi (okres adolescencji i wczesnej dorosłości). Przykładem są osiągnięcia Evarista Galois'a, Srinivasa Aiyangar'a Ramanujana, Terence'a Tao. Ma to swoje biologiczne podłoże jak twierdzili M. Spitzer (2012) i D.A. Kramer (2003) w strategii rozumowania. Najpierw jednak musiał pojawić się jakiś imperatyw w kierunku rozwoju uzdolnień. Celem prezentacji będzie przedstawienie wszystkich wydarzeń przełomowych w życiu zdolnych adolescentów i młodych dorosłych, które miały związek z ukierunkowaniem ich umysłów w stronę matematyki.

Andrzej WIŚNICKI

Pedagogical University of Krakow (Cracow, Poland)

Linear and nonlinear extensions of the Ryll-Nardzewski theorem

The Ryll-Nardzewski theorem asserts–in its basic and best-known form–that if K is a nonempty weakly compact convex subset of a Banach space, then any semigroup of affine isometries of K has a common fixed point. A general version is much stronger and concerns the distal affine systems on locally convex topological spaces.

In this talk we show a few extensions of this theorem, both linear and nonlinear, and discuss their applications in operator theory and geometric group theory. In particular, we show how to apply the Bader–Gelander–Monod theorem [1] concerning L-embedded Banach spaces to give a direct proof of the derivation problem for group algebras studied since 1960s, and take a new look at the Kadison similarity problem (see [3]) by applying the Lang fixed point theorem [2] for isometries in L^∞. We also present fully nonlinear extensions of the Ryll-Nardzewski theorem proved recently in [6].

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Alicja WOLNY-DOMINIAK

University of Economics in Katowice (Katowice, Poland)

Risk premium in property/casualty insurance - statistical approach

Generalised Linear Model (GLM) is a popular statistical tool now used by insurance companies. The model is most often applied in risk assessment for short-term non-life insurance schemes generating mass risk portfolios and in loss reserve prediction, particulary in *ratemaking* and *loss (claim) reserving*. Due to the insurance data specificity and considering the progress made in computational techniques as well as the growing amount of information gathered by insurers, different kinds of modifications of GLMs are now in practical use. Zero-inflated General Poisson claim frequency, overdispersion or heavy-tailed empirical claim severity distribution are the examples here. In the first part of the lecture a brief overview over the GLMs and their modifications used in ratemaking will be provided. After that Tweedie family of distributions applicable in the modeling of claim severity will be considered in detail. Next a variety of models for the count variable, which is the number of claims are presented. Considerations about the goodness of fit of the model (AIC, BIC, deviance) and the method of comparing models (cross-validation, bootstrapping) will end this part.

Nowadays the typical situation is to cover several LOBs in one risk, as e.g. automobile risk can be split into TPL (third part liability), MOD (motor own damage), fire, theft and so on. In this case, quantile premium, which captures the dependency among LOBs, can improve in practice the determination of risk-based capital requirements for P&C insurers, setting overall risk target by senior management, pricing of excess-of-loss reinsurance treaties or designing scenario analyses, to name a couple of applications. In the second part of the lecture the copula-based regression model dedicated to estimate the quantile premium for single risk in insurance covering dependent LOBs will be presented. To obtain the premium, first the dependency between LOBs is captured by the copula and second the Monte Carlo simulation is performed. Finally, some properties of quantile premium will be analyzed. The lecture will be finish by considerations about the new approach in ratemaking, which is telematrics.

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Wiesław WÓJCIK

Uniwersytet Humanistyczno-Przyrodniczy im. Jana Długosza w Częstochowie
(Częstochowa, Poland)

Uniwersalny charakter badań matematycznych Stefana Mazurkiewicza

W pracy chciałbym ukazać Stefana Mazurkiewicza, współtwórcę warszawskiej szkoły matematycznej, jako matematyka uniwersalnego, który wykraczał poza specyfikę warszawskiej szkoły matematycznej. Pokażę, że jego rozumienie uniwersalności matematyki odbiegało od tego, które było podstawą programu badawczego Zygmunta Janiszewskiego. Widoczne jest to w zagadnieniach, które podejmował i w sposobie ich rozwiązywania. Imponujący jest zakres obszarów badawczych. Poza badaniami w zakresie topologii geometrycznej (charakterystyki kontinuów nierozkładalnych), zajmował się teorią szeregów i jej zastosowaniami (np. do hydrodynamiki) oraz teorią funkcji analitycznych. Miał znaczące wyniki w zakresie badań podstaw matematyki, teorii wymiaru (badania topologicznego pojęcia wymiaru i definicja małego wymiaru indukcyjnego) oraz teorii prawdopodobieństwa. Sformułował własny program badawczy w zakresie badań podstaw rachunku prawdopodobieństwa. Udowodnił mocne prawo wielkich liczb (niezależnie od F. Cantelliego) oraz podał aksjomatykę rachunku prawdopodobieństwa ważną dla badań matematycznych, jak i filozoficznych. Nie można też zapominać o jego wkładzie w rozwój kryptografii. Został powołany w czasie wojny polsko-bolszewickiej jako współpracownik w Biurze

Szyfrów przy Sztabie Generalnym WP. Miał istotny udział w rozszyfrowaniu sowieckich komunikatów wojennych, przez cały okres międzywojenny był ekspertem w zakresie kryptografii i prowadził zajęcia na tajnym kursie kryptograficznym (na który uczęszczali, między innymi, późniejsi pogromcy Enigmy). Mazurkiewicz miał też zainteresowania poza matematyczne. Był znawcą i miłośnikiem historii oraz literatury pięknej, a także poetą. W roku 1927 wydał (wraz z M. Duninem) Sonety patetyczne.

Małgorzata ZAMBROWSKA

Akademia Pedagogiki Specjalnej (Warszawa, Poland)

Geometria w pierwszych latach szkolnej edukacji. Wybrane programy nauki z lat 1792 – 2020

„Geometria w pierwszych latach szkolnej edukacji. Wybrane programy nauki z lat 1792 – 2020" W pierwszych latach nauki w szkole jest obecnie niewiele treści związanych z geometrią. Uczniowie kończący I etap edukacyjny w roku 2020, zgodnie z obowiązującą podstawą programową, powinni znać nazwy niektórych figur, dokonywać pomiarów długości, rozpoznawać figury symetryczne i to właściwie wszystko. W referacie zostaną omówione wybrane programy nauki, szkolne podręczniki i poradniki metodyczne z lat 1792-2020 pod kątem zawartych w nich treści geometrycznych dla młodszych lat nauki. Najwięcej treści geometrycznych zawierały programy nauki z lat 20 XX w. Jednak geometria umieszczona była wówczas nie tylko w programie matematyki, ale też rysunków. Uczniowie w początkowych latach poznawali figury płaskie i ich własności, rysowali proste prostopadłe, równoległe, kąty o różnych rozwartościach i figury płaskie. Obliczali pola powierzchni i objętości. Programy te zostały zmienione już po 10 latach i geometria niemal całkowicie zniknęła z pierwszych lat szkolnej edukacji na długie dziesięciolecia. Aleksander Karp przestrzega przed uleganiem mitowi „błogosławionej przeszłości" w badaniach dotyczących historii nauczania matematyki. Pokażę, że jeśli chodzi o nauczanie geometrii przeszłość rzeczywiście nie była „błogosławiona" – dawniej nie zawsze było lepiej z nauczaniem geometrii we wczesnych latach nauki.

Słowa kluczowe: historia nauczania, edukacja matematyczna, geometria, dydaktyka.

Agnieszka M. ZBRZEZNY[1], Andrzej ZBRZEZNY[2]

[1]*Faculty of Mathematics and Computer Science, University of Warmia and Mazury in Olsztyn* (Olsztyn, Poland)

[2]*Institute of Mathematics and Computer Science, Jan Długosz University in Częstochowa* (Częstochowa, Poland)

Checking MTL Properties of Timed Automata with Dense Time using Satisfiability Modulo Theories

We investigate a new SMT-based bounded model checking (BMC) method for the existential part of Metric Temporal Logic (MTL). The MTL logic is interpreted over linear dense infinite time models generated by timed automata with dense time. We implemented the new SMT-based bounded model checking technique for MTL and as a case study we applied the technique

to the analysis of the Timed Generic Pipeline Paradigm and Timed Train Controller System, both modelled by a network of timed automata.

LIST OF PARTICIPANTS

Molla Ahamed
Indian Institute of Technology Bhubaneswar, India
mba15@iitbbs.ac.in

Om Ahuja
Kent State University, USA
oahuja@kent.edu

Vasudeva Rao Allu
Indian Institute of Technology Bhubaneswar, India
avrao@iitbbs.ac.in

Swati Anand
University of Delhi, India
swati_anand01@yahoo.com

Sairam Kaliraj Anbareeswaran
Indian Institute of Technology Ropar, India
sairam@iitrpr.ac.in

Piotr Artiemjew
University of Warmia and Mazury in Olsztyn, Poland
artem@matman.uwm.edu.pl

Józef Banaś
Rzeszów University of Technology, Poland
jbanas@prz.edu.pl

Martina Bečvářová
Czech Technical University in Prague, Czech Republic
becvamar@fd.cvut.cz

Subzar Beig
University of Kashmir, India
beighsubzar@gmail.com

Gabriela Białoskórska
Cracow University of Technology, Poland
gabrielabialoskorska@icloud.com

Piotr Błaszczyk
Pedagogical University of Krakow, Poland
pb@up.krakow.pl

Agnieszka Bojarska-Sokołowska
University of Warmia and Mazury in Olsztyn, Poland
bojarska@matman.uwm.edu.pl

Daoud Bshouty
Technion, Israel Institute of Technology, Israel
daoud@technion.ac.il

Nak Eun Cho
Pukyong National University, Korea
necho@pknu.ac.kr

Jacek Chudziak
University of Rzeszów, Poland
chudziak@ur.edu.pl

Ihor Chyzhykov
University of Warmia and Mazury in Olsztyn, Poland
chyzhykov@matman.uwm.edu.pl

Zoltán Ernő Csajbók
University of Debrecen, Hungary
csze.conf@gmail.com

Ewa Dąbrowska
Gdynia Maritime University, Poland
ebnya@wp.pl

K Dhurai
Government College of Engineering, Dharmapuri; Anna University, Chennai, Tamil Nadu, India
dhuraimaths@gmail.com

Renata Długosz
Lodz University of Technology, Poland
renata.dlugosz@p.lodz.pl

Anna Dobosz
University of Warmia and Mazury in Olsztyn, Poland
dobosz@uwm.edu.pl

Stanisław Domoradzki
University of Rzeszów, Poland
domoradz@ur.edu.pl

Jacek Dziok
University of Rzeszów, Poland
jdziok@ur.edu.pl

Mark Elin
ORT Braude College, Israel
mark_elin@braude.ac.il

Beata Fałda
State School of Higher Education in Chełm, Poland
beatafalda@gmail.com

Boyana Garkova
South-West University, Bulgaria
big@swu.bg

Marek Golasiński
University of Warmia and Mazury in Olsztyn, Poland
marekg@matman.uwm.edu.pl

Przemysław Górecki
University of Warmia and Mazury in Olsztyn, Poland
pgorecki@matman.uwm.edu.pl

Himadri Halder
Indian Institute of Technology Bhubaneswar, India
hh11@iitbbs.ac.in

Aleksandra Huczek
Pedagogical University of Krakow, Poland
huczek.aleksandra@gmail.com

Jan Jakóbowski
University of Warmia and Mazury in Olsztyn, Poland
jjakob@matman.uwm.edu.pl

Piotr Jastrzębski
University of Warmia and Mazury in Olsztyn, Poland
piotr.jastrzebski@uwm.edu.pl

Izabela Jóźwik
Lodz University of Technology, Poland
ijozwik@p.lodz.pl

Sivasubramanian Kavitha
SDNB Vaishnav College for Women, Chennai, Tamilnadu, India
kavi080716@gmail.com

Adel Khalfallah
King Fahd University of Petroleum and Minerals, Kingdom of Saudi Arabia
khelifa@kfupm.edu.sa

Aleksandra Kiślak-Malinowska
University of Warmia and Mazury in Olsztyn, Poland
akis@uwm.edu.pl

Mikhail Kolev
University of Warmia and Mazury in Olsztyn, Poland
kolev@matman.uwm.edu.pl

Krzysztof Kołowrocki
Gdynia Maritime University, Poland
k.kolowrocki@wn.umg.edu.pl

Jan Koroński
Cracow University of Technology, Poland
jan.koronski@gmail.com

Bogumiła Kowalczyk
University of Warmia and Mazury in Olsztyn, Poland
b.kowalczyk@matman.uwm.edu.pl

Mateusz Krukowski
Lodz University of Technology, Poland
mateusz.krukowski@p.lodz.pl

Tomasz Krzywicki
University of Warmia and Mazury in Olsztyn, Poland
tomasz.krzywicki@matman.uwm.edu.pl

Sushil Kumar
Bharati Vidyapeeth's College of Engineering, GGS IP University, New Delhi, India
sushilkumar16n@gmail.com

Radosław Kycia
Masaryk University; Cracow University of Technology, Czechia, Poland
kycia.radoslaw@gmail.com

Mirosław Lachowicz
University of Warsaw, Poland
M.Lachowicz@mimuw.edu.pl

Adam Lecko
University of Warmia and Mazury in Olsztyn, Poland
alecko@matman.uwm.edu.pl

Millenia Lecko
Rzeszów University of Technology, Poland
mlecko@prz.edu.pl

See Keong Lee
Universiti Sains Malaysia, Malaysia
sklee@usm.my

Piotr Liczberski
Lodz University of Technology, Poland
piotr.liczberski@p.lodz.pl

Adam Ligęza
University of Warsaw, Poland
a.ligeza@mimuw.edu.pl

Monika Lindner
Lodz University of Technology, Poland
mlindner@p.lodz.pl

Abdallah Lyzzaik
American University of Beirut, Lebanon
alyzzaik@gmail.com

S Madhumitha
National Institute of Technology, Trichy, India
416119002@nitt.edu

Beata Magryta
Gdynia Maritime University, Poland
b.magryta@wn.umg.edu.pl

Wiesław Majchrzak
Lodz University of Technology, Poland
wieslaw.majchrzak@p.lodz.pl

Somya Malik
National Institute of Technology, Tiruchirappalli, Tamil Nadu, India
arya.somya@gmail.com

Jacek Marchwicki
University of Warmia and Mazury in Olsztyn, Poland
marchewajaclaw@gmail.com

Miodrag Mateljević
University of Belgrade, Serbia
miodrag@matf.bg.ac.rs

Ivan Matychyn
University of Warmia and Mazury in Olsztyn, Poland
matychyn@matman.uwm.edu.pl

Piotr Michalak
Gdynia Maritime University, Poland
p.michalak@wn.umg.edu.pl

Irina Naskinova
South-West University "Neofit Rilski", Bulgaria
naskinova@swu.bg

Iveta Nikolova
University of Architecture, Civil Engineering and Geodesy, Bulgaria
iveta.nikolova@abv.bg

Maria Nowak
Maria Curie-Skłodowska University in Lublin, Poland
mt.nowak@poczta.umcs.lublin.pl

Viktoriia Onyshchenko
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Ukraine
oviva@ukr.net

Krzysztof Pancerz
State School of Higher Education in Zamosc, Poland
krzysztof.pancerz@upz.edu.pl

Abhishek Pandey
Indian Institute of Technology Bhubaneswar, India
ap57@iitbbs.ac.in

Maciej Parol
The John Paul II Catholic University of Lublin, Poland
mparol@kul.pl

Dariusz Partyka
The John Paul II Catholic University of Lublin, Poland
partyka@kul.lublin.pl

James F. Peters
University of Manitoba; Adiyaman University, Canada
james.peters3@ad.umanitoba.ca

Saminathan Ponnusamy
Indian Institute of Technology Madras, India
samy@iitm.ac.in

Andrzej Poszwa
University of Warmia and Mazury in Olsztyn, Poland
poszwa@matman.uwm.edu.pl

V Ravichandran
National Institute of Technology, Tiruchirappalli, India
ravic@nitt.edu

Kanaga Rethinasabapathi
National Institute of Technology, Trichy., India
416118052@nitt.edu

Swadesh Sahoo
Indian Institute of Technology Indore, Indore, India
swadesh.sahoo@iiti.ac.in

Ken-ichi Sakan
Osaka City University, Japan
ksakan@sci.osaka-cu.ac.jp

Asha Sebastian
National Institute of Technology, Tiruchirappalli, Tamil Nadu, India
ashanitt18@gmail.com

Kanika Sharma
Atma Ram Sanatan Dharma College, University of Delhi, India
kanika.divika@gmail.com

Meghna Sharma
University of Delhi, India
meghnasharma203@gmail.com

David Shoikhet
Holon Institute of Technology, Israel
davidsho@hit.ac.ild

Young Jae Sim
Kyungsung University, Korea
yjsim@ks.ac.kr

Sanjeev Singh
Indian Institute of Technology Indore, India
snjvsngh@iiti.ac.in

Srikandan Sivasubramanian
University College of Engineering Tindivanam Anna University Tamilnadu India, India
sivasaisastha@rediffmail.com

Bogdan Staruch
University of Warmia and Mazury in Olsztyn, Poland
bstar@uwm.edu.pl

Bożena Staruch
University of Warmia and Mazury in Olsztyn, Poland
bostar@matman.uwm.edu.pl

Toshiyuki Sugawa
Tohoku University, Japan
sugawa@math.is.tohoku.ac.jp

Marek Svetlik
University of Belgrade, Serbia
svetlik@matf.bg.ac.rs

Anbhu Swaminathan
Indian Institute of Technology, Roorkee, India
a.swaminathan@ma.iitr.ac.in

Anna Szpila
University of Rzeszów, Poland
anszpila@ur.edu.pl

Jacek Szypulski
University of Warmia and Mazury in Olsztyn, Poland
uwm@szypulski.it

Barbara Śmiarowska
University of Warmia and Mazury in Olsztyn, Poland
b.smiarowska@matman.uwm.edu.pl

Sławomir Tadeja
Immersive Toolbox, Poland
stadeja@immersivetoolbox.com

Małgorzata Terepeta
Lodz University of Technology, Poland
malgorzata.terepeta@p.lodz.pl

Derek K. Thomas
Swansea University, United Kingdom
d.k.thomas@swansea.ac.uk

Mateusz Torbicki
Gdynia Maritime University, Poland
m.torbicki@wn.umg.edu.pl

Katarzyna Trąbka-Więcław
Lublin University of Technology, Poland
k.trabka@pollub.pl

Edyta Trybucka
University of Rzeszów, Poland
eles@ur.edu.pl

Nikola Tuneski
Ss. Cyril and Methodius University, Republic of North Macedonia
nikola.tuneski@mf.edu.mk

Wojciech Walat
University of Rzeszów, Poland
walat@ur.edu.pl

Lateef Ahmad Wani
Indian Institute of Technology Roorkee, India
lateef17304@gmail.com

Maja Wenderlich
The Maria Grzegorzewska University, Poland
mwenderlich@aps.edu.pl

Andrzej Wiśnicki
Pedagogical University of Krakow, Poland
andrzej.wisnicki@up.krakow.pl

Alicja Wolny-Dominiak
University of Economics in Katowice, Poland
woali@ue.katowice.pl

Wiesław Wójcik
Jan Dlugosz University in Czestochowa, Poland
w.wojcik@ujd.edu.pl

Józef Zając
State School of Higher Education in Chełm, Poland
jzajac.stany@gmail.com

Małgorzata Zambrowska
Maria Grzegorzewska University, Poland
m.zambrowska@gmail.com

Paweł Zaprawa
Lublin University of Technology, Poland
p.zaprawa@pollub.pl

Mykhaylo Zarichnyy
University of Rzeszów, Poland
zarichnyi@yahoo.com

Agnieszka Zbrzezny
University of Warmia and Mazury in Olsztyn, Poland
agnieszka.zbrzezny@matman.uwm.edu.pl